## 0.12 Solving Systems of Equations with Matrices

To solve a system of linear equations using matrices on the calculator, we must

- Enter the augmented matrix.
- Specify the elementary row operations we wish to perform, one operation at a time.

  - or, if allowed by your instructor -

  We can go directly to row-echelon or reduced row-echelon form.

Let's solve this system with the augmented matrix shown:

$$\begin{cases} 3x + 6y + 5z &= -13 \\ 4x + 8y - 4z &= 4 \\ -2x + y + 12z &= -17 \end{cases} \qquad \begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Note: If it is possible the matrices will involve fractions, be sure to indicate in the *Mode Setting* for `Float` the number of decimal places you wish to display.

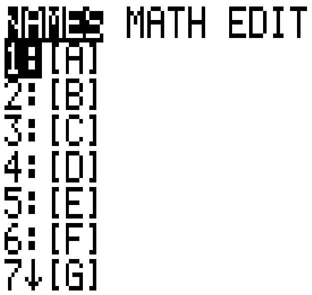
TI-83 (see page 164), TI-89 (see page 172), TI-86 (see page 181).

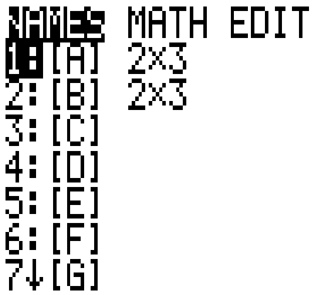
# TI-83: Matrices

In the directions given, TI-83 Plus users press 2nd and Matrx as shown, but the TI-83 users press only Matrx.

## Enter the augmented matrix

Press 2nd and Matrx, where `Matrx` is the second operation on the $x^{-1}$ key, to see the left screen shown. If you see something like the right screen, you already have matrices defined.

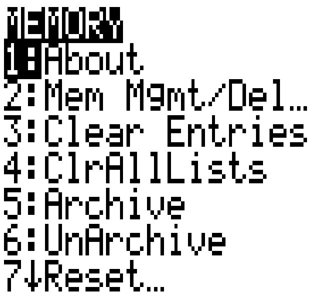


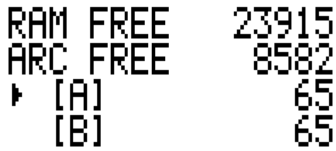

If you do not need to delete any matrices, continue on page 165.

## Deleting matrices previously defined

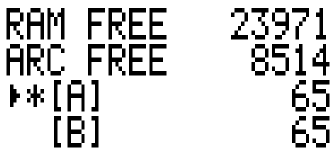
To erase the matrices already defined, press 2nd and Mem, where `Mem` is the second function on the + key, to see the left screen shown. Then select `2: Mem Mgmt/Del...` to see the middle screen shown, and then select `5: Matrix...` to see the right screen .

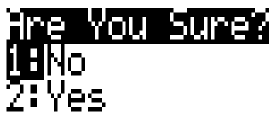







With the arrow beside [A], press Enter and an asterisk appears beside [A] (left screen). Press Del. When asked *Are you sure?* (middle screen), choose `2:Yes`. We now see that matrix [A] has been deleted (right screen).

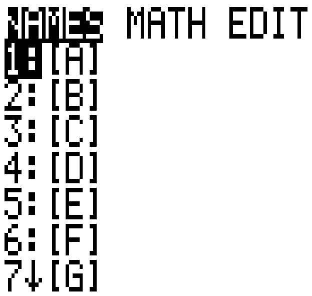


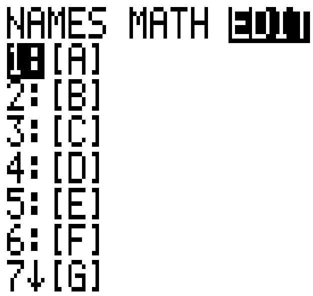




Repeat the process for matrix [B], and any others you wish to delete.
To exit this part, press 2nd and Quit

## Enter the augmented matrix continued

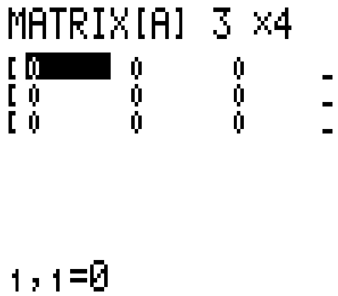
If you need to get the left screen again, press 2nd and Matrx. With the right arrow, move to the `Edit` option (middle screen). Select the matrix to define by using the arrow key, or by entering the number beside the matrix name. To select matrix [A], press 1. Result is shown on the right.



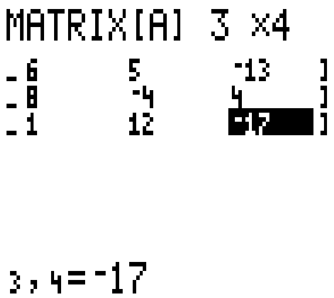


We need to specify the dimensions of the matrix, and the value of each element in the matrix. We have a $3 \times 4$ matrix to enter, 3 rows and 4 columns. Type 3 and press Enter, then type 4 and press Enter. Notice the row and column designation in the bottom left corner. This shows we are currently at row 1, column 1, and the current value of that element is zero.




The rows and columns of the matrix are set up ready for us to enter the particular element values. Type each value from the augmented matrix, pressing Enter after each entry.

$$\begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

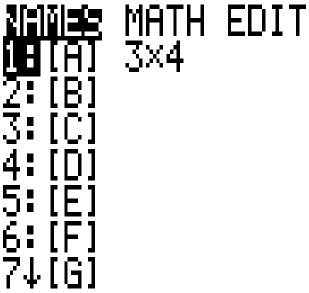



Notice that only 3 columns are displayed at one time in the calculator window. We can move from element to element, row to row, column to column with the arrow keys at any time. We are currently seeing columns 2, 3, and 4 in the matrix shown above on the right. Notice, the bottom left corner shows the cursor's current position, `3, 4=-17`, is row 3, column 4, with a value of −17.

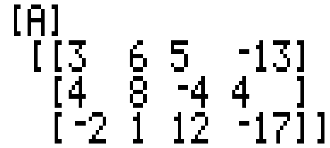
We need to exit the `Edit` mode in order to perform our matrix operations. Press 2nd and Quit.

To see matrix [A] that we just entered, press (2nd) and (Matrx) to see the named matrices (left screen). Press (Enter) to select matrix [A], and we get the middle screen showing only the name [A]. Press (Enter) again to see the right screen.








## Perform elementary row operations

We have four choices on the calculator, same as the rules we are given algebraically.

- Swap two rows, called `rowSwap`.
  Example: `rowSwap([A],2,3)` swaps rows 2 and 3 in matrix [A].

- Add two rows, called `row+(`.
  Example: `row+([A],1,2)` adds row 1 and 2, placing the result in row 2.

- Multiply one row by a nonzero constant, called `*row(`.
  Example: `*row(-5,[A],2)` multiples row 2 by $-5$, placing the result in row 2.

- Multiply one row by a nonzero constant and add that result to another row, called `*row+(`.
  Example: `*row+(-5,[A],3,2)` multiplies row 3 by $-5$, adds the result to row 2, placing the result in row 2.

None of these operations will *save* the resulting matrix. We have to save the new matrix by using the `Store` command. Continue reading for directions.

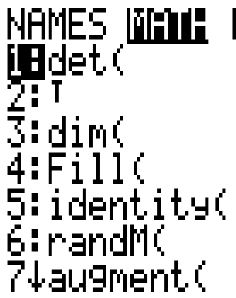
To see how to use each of these elementary row operations, we will do the appropriate steps to put the given augmented matrix into row-echelon form.

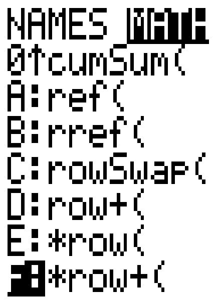
In the pictures on the next few pages, you will notice that as we do each operation on the matrix, the calculator screen scrolls, showing a portion of the previous matrix, the command given, and the new matrix. After a few screens, you will get used to finding the new matrix and ignoring the old work.

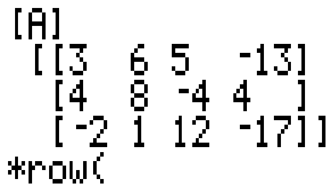
When performing elementary row operations, it is possible that something like `1E-13` may be used as an approximation for zero due to the way the calculator performs calculations. If this occurs, the number should be treated as zero. It is strongly suggested that you return to the Matrix Edit screen and replace the entry of `1E-13` with a 0.

To see the augmented matrix we are using, look at the right screen below.
Let's first multiply each element in row 2 by $\frac{1}{4}$, so we create a leading entry of 1 for row 2.

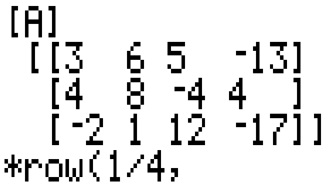
Press 2nd and Matrx. Use the right arrow key to highlight `Math` (see left screen). Press the up arrow key once to see more operations available (see middle screen). Select `E:*row(` by using the up arrow to highlight the entry `E:*row(` and press Enter. The right screen shows the command written below matrix [A].

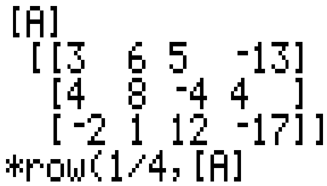


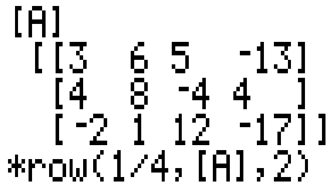



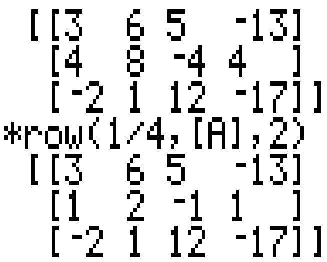

To multiply row 2 by $\frac{1}{4}$, we need to enter `*row(1/4,[A],2)` so enter `1/4` followed by a comma (see the left screen). Specify the matrix by pressing 2nd and Matrx, then select matrix [A] by pressing 1 or Enter, and we have the middle screen. Type `,2)` to finish the command. We are saying to multiply row 2 in matrix [A] by $\frac{1}{4}$, and replace row 2 with the result. It should look like the right screen.

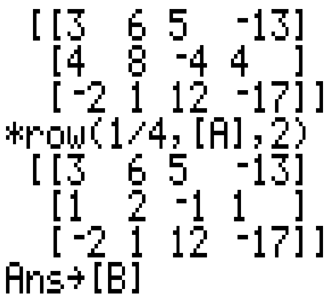


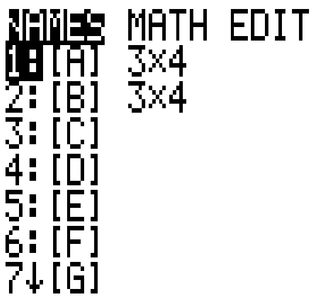



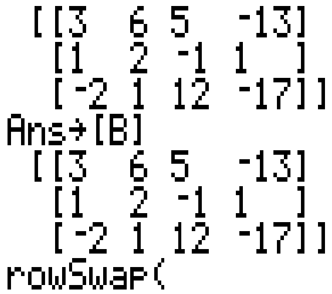

Press Enter to see the new matrix. As expected, we now have the left screen. However, the calculator does not save this new matrix. Since we want to do more to it, we have to save it for further use. Do so by pressing Sto→, then press 2nd and Matrx, and select matrix [B] by pressing 2. Now, we see the middle screen, which says to store our answer in matrix [B]. Press Enter and we see matrix [B] repeated (not shown here). However, if we now look at the list of matrices, press 2nd and Matrx, we see that matrix [B] is defined. To get rid of the list of matrices and return to our problem, press clear.

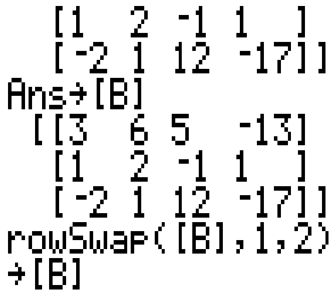


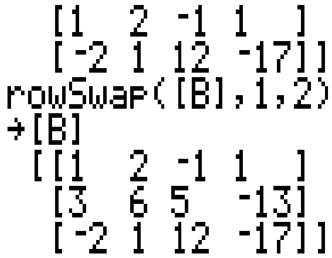



(To see matrix [B] on this page, look at the middle screen below).
Since we have a 1 as the leading entry of row 2, let's swap row 1 and row 2. Press 2nd and Matrx, move to `Math`, hit the up arrow once to see the menu options (left screen). Select `C:rowSwap(` and we see the middle screen. To swap rows 1 and 2 in matrix [B], we first tell what matrix to use, and then what rows to swap. Press 2nd and Matrx, and select 2 for matrix [B]. Finish typing the command `,1,2)` so it looks like `rowSwap([B],1,2)`, but don't hit Enter yet! Let's also save our new matrix and place it back into matrix [B] to save a little time. Press Sto→, press 2nd and Matrx, and choose matrix [B] again (or some other name of your choosing). Now we see the right screen.

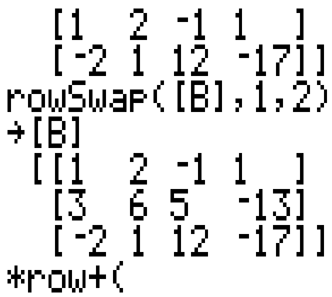


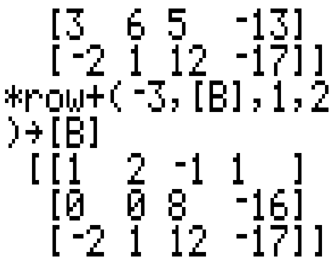



Press Enter and we have this screen. Notice the leading entry of row 1 is 1, just as we wanted.




Now, let's turn the 3 in row 2 into 0. Multiply row 1 by $-3$ and add it to row 2, creating a new row 2. To do this, we need to give the command `F:*row+(−3,[B],1,2)` and store the result in [B]. Press 2nd and Matrx, go to `Math` and use the up arrow to get more options. This time select `F:*row+(` and we have the left screen. We need to enter $-3, [B], 1, 2)$ so type $-3,$ then enter [B] by using 2nd and Matrx, and entering 2 to select [B]. Type the `,1,2)` to finish the expression. This says, in matrix [B], multiply the first row specified (row 1) by $-3$, add the result to the second row specified (row 2), and place the final result back in the second row specified (row 2). Now store this answer in matrix [B] by using Sto→, 2nd and Matrx, choosing [B]. After we press Enter to execute the command, we see the right screen.





(To see our current matrix, look at the matrix in the left screen below.)

To turn the $-2$ in row 3 into 0, we need to multiply row 1 by 2, add the result to row 3 and replace row 3. We need to enter `*row+(2,[B],1,3)` and store the result in [B].

Press [2nd] and [Matrx], move to `Math`, go up once, and select `F:*row+(` to see the left screen. Enter `2,[B],1,3)` and store the matrix back into [B], to see the middle screen. After pressing [Enter], we see the right screen.

```
  [3  6 5  -13]
  [-2 1 12 -17]]
*row+(-3,[B],1,2
)→[B]
 [[1  2 -1 1  ]
  [0  0 8  -16]
  [-2 1 12 -17]]
*row+(
```

```
  [-2 1 12 -17]]
*row+(-3,[B],1,2
)→[B]
 [[1  2 -1 1  ]
  [0  0 8  -16]
  [-2 1 12 -17]]
*row+(2,[B],1,3)
→[B]
```

```
  [0  0 8  -16]
  [-2 1 12 -17]]
*row+(2,[B],1,3)
→[B]
 [[1 2 -1 1  ]
  [0 0 8  -16]
  [0 5 10 -15]]
```

Next, multiply row 3 by $\frac{1}{5}$ to get our leading entry of 1 in row 3. We need to enter `E:*row(1/5,[B],3)` and store the result in [B].

Directions if needed: Press [2nd] and [Matrx], move to `Math`, go up once, and select `E:*row(`. Enter `1/5,` and indicate matrix [B] by pressing [2nd] and [Matrx] and selecting [B]. Finish typing `,3)` and store it in matrix [B] by pressing [Sto→] then [2nd] and [Matrx], selecting [B], and we see the left screen. Press [Enter] to see the right screen.

```
  [-2 1 12 -17]]
*row+(2,[B],1,3)
→[B]
  [[1 2 -1 1  ]
   [0 0 8  -16]
   [0 5 10 -15]]
*row(1/5,[B],3)→
[B]
```

```
  [0 0 8  -16]
  [0 5 10 -15]]
*row(1/5,[B],3)→
[B]
 [[1 2 -1 1  ]
  [0 0 8  -16]
  [0 1 2  -3 ]]
```

To finish placing the matrix into row-echelon form, we need to change the 8 in row 2 to a 1 by multiplying row 2 by $\frac{1}{8}$.

We need to enter `E:*row(1/8,[B],2)`

Directions: Press [2nd] and [Matrx], move to `Math`, go up once, and select `E:*row(`. Enter `1/8,` and indicate matrix [B] by pressing [2nd] and [Matrx], select [B], finish typing `,2)` and store it in matrix [B] by pressing [Sto→], then [2nd] and [Matrx], selecting [B], and we see the left screen. Press [Enter] to see the middle screen. An unnecessary step we could add would be to swap rows 2 and 3. Do so by entering `C:rowSwap([B],2,3)`, storing it in matrix [B] if you wish, and we see the right screen.

```
   [0 5 10 -15]]
*row(1/5,[B],3)→
[B]
  [[1 2 -1 1  ]
   [0 0 8  -16]
   [0 1 2  -3 ]]
*row(1/8,[B],2)→
[B]
```

```
   [0 0 8  -16]
   [0 1 2  -3 ]]
*row(1/8,[B],2)→
[B]
  [[1 2 -1 1 ]
   [0 0 1  -2]
   [0 1 2  -3]]
```

```
   [0 0 1  -2]
   [0 1 2  -3]]
rowSwap([B],2,3)
→[B]
  [[1 2 -1 1 ]
   [0 1 2  -3]
   [0 0 1  -2]]
```

We have our row-echelon form of the matrix. The system of equations now looks like

$$\begin{cases} 1x + 2y - 1z &= 1 \\ 1y + 2z &= -3 \\ 1z &= -2 \end{cases}$$

By using back substitution, we can now finish solving the system of equations.

If the instructor allows on some problems, you may go straight from the augmented matrix to the row-echelon form of the matrix by doing the following.

After you have entered the augmented matrix [A] (see left screen), go to the matrix options menu by pressing [2nd] and [Matrx], use the up arrow once. Select `A:ref(` which stands for *row-echelon form.* Enter matrix [A] and close the parentheses. Store the answer if you wish in, say matrix [C], so it looks like the middle screen. Press [enter] to see the final result (the right screen).

```
[A]
 [[3  6 5  -13]
  [4  8 -4 4  ]
  [-2 1 12 -17]]
```

```
[A]
 [[3  6 5  -13]
  [4  8 -4 4  ]
  [-2 1 12 -17]]
ref([A])→[C]
```

```
 [[3  6 5  -13]
  [4  8 -4 4  ]
  [-2 1 12 -17]]
ref([A])→[C]
  [[1 2 -1 1 ]
   [0 1 2  -3]
   [0 0 1  -2]]
```

Another option we can use, if the instructor allows, is the *reduced row-echelon form* command. This takes the augmented matrix and places it into reduced row-echelon form immediately.

Again after you have entered the augmented matrix [A] (see left screen), go to the matrix options menu by pressing 2nd and Matrx, use the up arrow once. Select `B:rref(` which stands for *reduced row-echelon form*. Enter matrix [A] and close the parentheses. There is probably no need to store this answer. So, our screen looks like the middle picture. Press Enter to see the final reduced row-echelon form of the matrix (the right screen).

```
[A]
 [[3  6 5  -13]
  [4  8 -4 4  ]
  [-2 1 12 -17]]
```

```
[A]
 [[3  6 5  -13]
  [4  8 -4 4  ]
  [-2 1 12 -17]]
rref([A])
```

```
  [[3  6 5  -13]
   [4  8 -4 4  ]
   [-2 1 12 -17]]
rref([A])
      [[1 0 0 -3]
       [0 1 0 1 ]
       [0 0 1 -2]]
```

From this last form, we can read our final answer: $x = -3, y = 1, z = -2$.

Press Clear to erase the screen.

These directions may look very complicated upon first reading. However, after you try them a few times and get used to what needs to be done, they will flow much easier and faster. Like most things, it does take practice to become fluent with the process.

# TI-89: Matrices

We are solving this system $\left\{\begin{array}{rcr} 3x+6y+5z & = & -13 \\ 4x+8y-4z & = & 4 \\ -2x+y+12z & = & -17 \end{array}\right.$

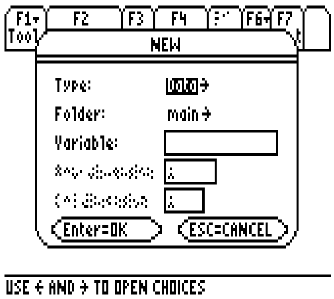
with an augmented matrix of $\left[\begin{array}{rrrr} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{array}\right]$

The printing on the calculator screen pictures is often very small, but you should be able to at least tell you are in the right place by what you see. (The printing is also very small on the calculator itself.) These directions use uppercase letters for the matrices to make the name more obvious. So you will see matrix A in the directions instead of matrix a which is shown on the calculator.

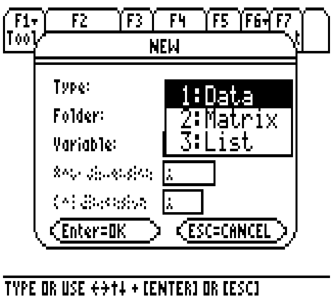
## Enter the augmented matrix

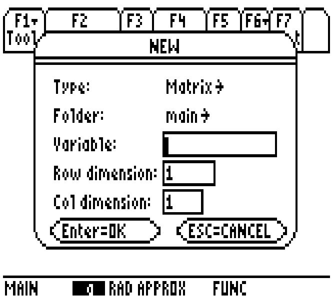
Press Apps to see the applications available (left screen). Press `6:Data/Matrix Editor` to see the editor options (middle screen). We want to define a *new* matrix, so select 3 (right screen).

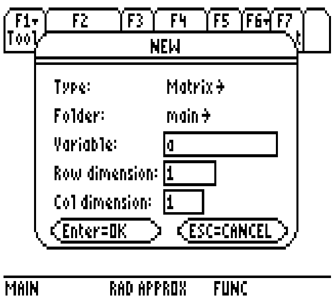





To specify what type of data, press the right arrow to see the options (left screen). Select `Matrix` (middle screen). With the arrow key, move down twice to the *variable* definition. Hold down Alpha and press A to name the matrix A (right screen).

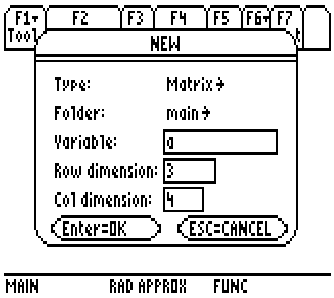


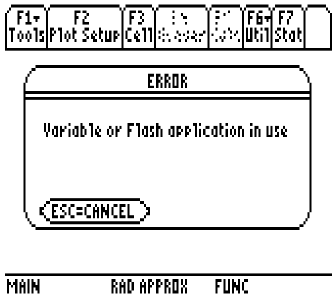



The row and column dimensions specify the size of the matrix we want to define.

Arrow down once more and enter the number of rows, which is 3. Arrow down once more and enter the number of columns, which is 4.(See left screen.) Press Enter twice to move to the editor so we can enter the elements of the matrix. If you already have a matrix named A, then you will get the message shown on the right screen.

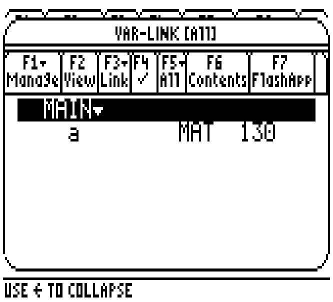


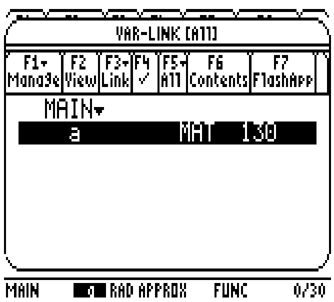



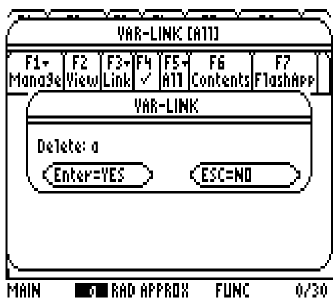

For those who have matrix A defined, press Esc and either choose another name for the matrix, or in our case, let's see how to delete the previous version of matrix A. Press Esc again to get back to the Home screen. If you do not have a matrix already defined, continue on page 174.

## Deleting previously defined matrices

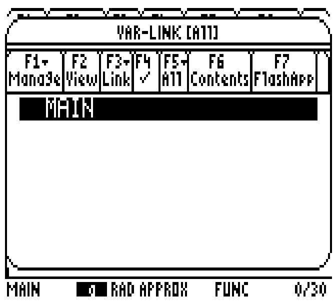
To erase the matrices already defined, press 2nd and Var-Link, where `Var-Link` is the second function on the - key to see the left screen shown. Arrow down to highlight the item you wish to delete (middle screen). Press ← to start the delete process and see the right screen.




Press Enter to say *yes* delete the matrix shown, and it is gone. If you have any others to delete, repeat the process.

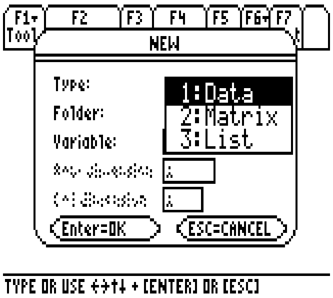


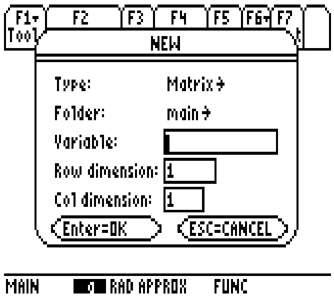

To exit this part, press Esc.

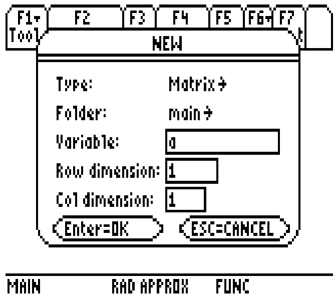
## Enter the augmented matrix continued

If needed, press Apps again, select 6 to get the Matrix Editor, and select 3 to start a new matrix.

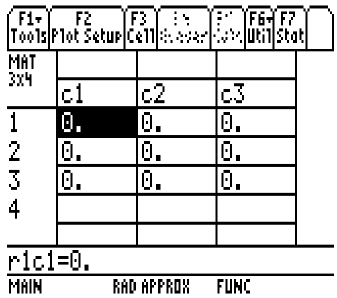
To specify what type of data, press the right arrow to see choices. Select `Matrix`. With the arrow key, move down twice to the *variable* definition. Hold down Alpha and press A to name the matrix A.



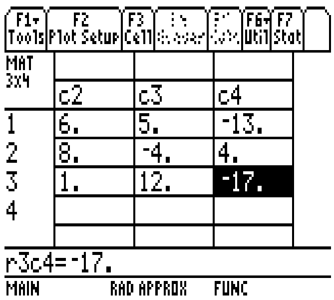




Arrow down once more and enter the number of rows, which is 3. Arrow down once more and enter the number of columns, which is 4. Press Enter twice to get to the matrix editor and the screen shown here.




The rows and columns of the matrix are set up ready for us to enter the particular element values. Type each value from the augmented matrix, pressing Enter after each entry.

$$\begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$




Notice that only 3 columns are displayed at one time in the calculator window. We can move from element to element, row to row, column to column with the arrow keys at any time. We are currently seeing columns 2, 3, and 4 in the picture above. Notice, the bottom left corner shows the cursor's current position, `r3c4`$= -17$, which is row 3, column 4, with a value of $-17$.

Now that the matrix is defined, we need to return to the *Home* screen to perform the elementary row operations. To exit the `Edit` mode, press Home.

## Perform elementary row operations

We have four choices on the calculator, same as the rules we are given algebraically.

- Swap two rows, called `rowSwap`.
  Example: `rowSwap(A,2,3)` swaps rows 2 and 3 in matrix A.

- Add two rows, called `rowAdd(`.
  Example: `rowAdd(A,1,2)` adds row 1 and 2, placing the result in row 2.

- Multiply one row by a nonzero constant, called `mRow(`.
  Example: `mRow(-5,A,2)` multiples row 2 by $-5$, placing the result in row 2.

- Multiply one row by a nonzero constant and add that result to another row, called `mRowAdd(`.
  Example: `mRowAdd(-5,A,3,2)` multiplies row 3 by $-5$, adds the result to row 2, placing the result in row 2.

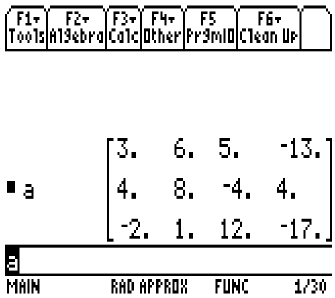
None of these operations will *save* the resulting matrix. We have to save the new matrix by using the `Store` command. Continue reading for directions.

To see how to use each of these elementary row operations, we will do the appropriate steps to put the given augmented matrix into row-echelon form.

In the pictures on the next few pages, you will notice that as we do each operation on the matrix, the calculator screen scrolls, showing a portion of the previous matrix, the command given, and the new matrix. After a few screens, you will get used to finding the new matrix and ignoring the old work.

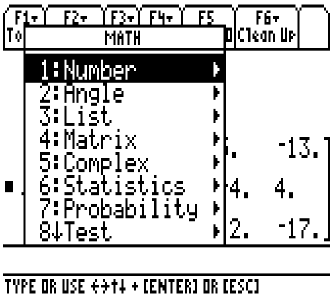
When performing elementary row operations, it is possible that something like `1E-13` may be used as an approximation for zero due to the way the calculator performs calculations. If this occurs, the number should be treated as zero. It is strongly suggested that you return to the Matrix Edit screen and replace the entry of `1E-13` with a 0.

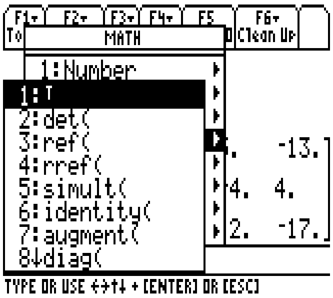
To see matrix A that we just entered, press Alpha and A. Press Enter.

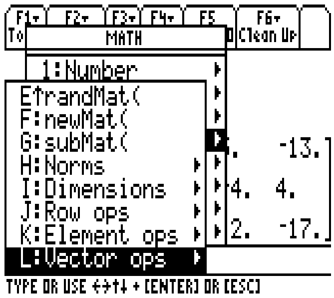


Let's first multiply each element in row 2 by $\frac{1}{4}$, so we create a leading entry of 1 for row 2.

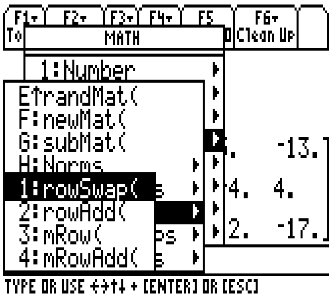
Press (2nd) and (Math), where `Math` is the second function on the (5) key, to see the left screen. Select `4:Matrix` and look at the options available (middle screen). Use the up arrow once to see more options (right screen).

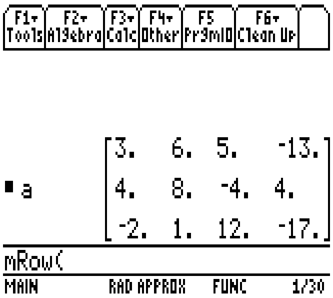


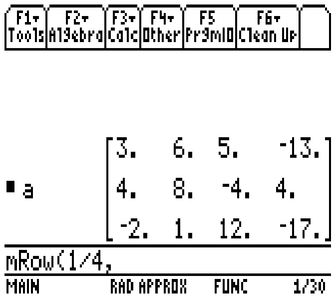



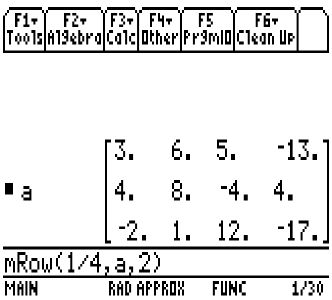

Now select `J:Row ops` to see the options available (left screen). Select `3:mRow(` and the command appears below our matrix (middle screen). To multiply row 2 by $\frac{1}{4}$, we need to enter `mRow(1/4,A,2)` so enter `1/4` followed by a comma (right screen).

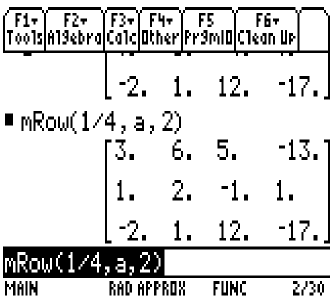


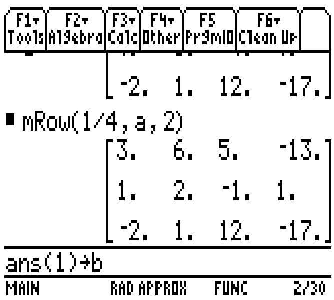




Now specify the matrix by pressing (Alpha) and (A), and finish typing `,2)` to finish the command. We are saying to multiply row 2 in matrix A by $\frac{1}{4}$, and replace row 2 with the result. It should look like the left screen. Press (Enter) to see the new matrix. As expected, we now have the middle screen. However, the calculator does not save this new matrix. Since we want to do more to it, we have to save it for further use. Do so by pressing (Sto→), then press (Alpha) and (B) to say store our answer in matrix B (right screen).

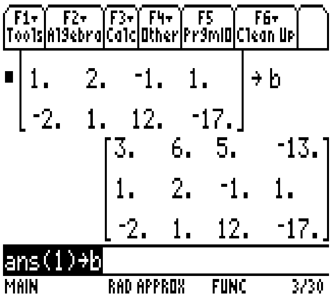


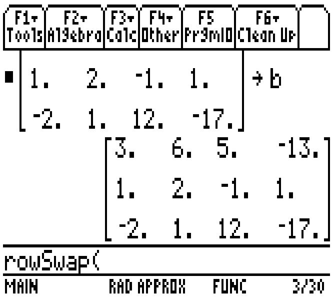



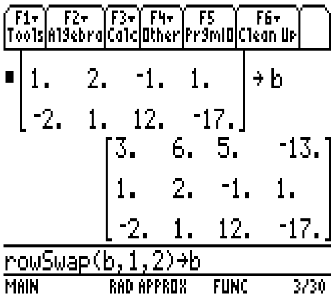



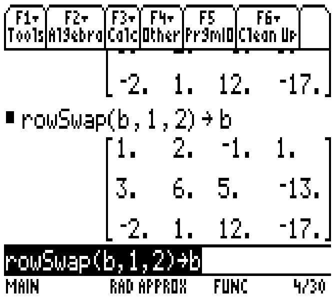
Press (Enter) to execute the command and we have the left screen below. Since we have a 1 as the leading entry of row 2, let's swap row 1 and row 2. We need to enter `rowSwap(B,1,2)` as our command. Press (2nd) and (Math), select `4:Matrix`, hit the up arrow once and select `J:Row ops`. Now select `1:rowSwap(` and we see the middle screen. To swap rows 1 and 2 in matrix B, we first tell what matrix to use, and then what rows to swap. Press (Alpha) and (B), then finish typing the command `,1,2)` but don't hit (Enter) yet! Let's also save our new matrix and place it back into matrix B to save a little time. Press (Sto→), press (Alpha) and (B) (or some other name of your choosing). Now we see the right screen.



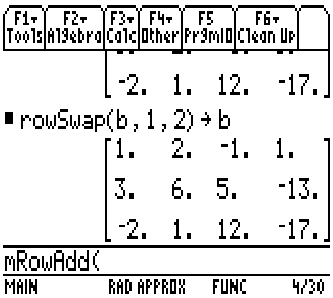


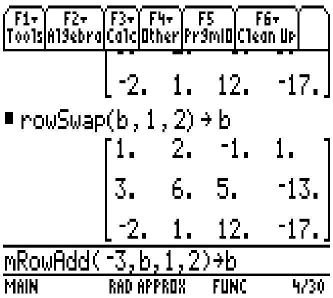


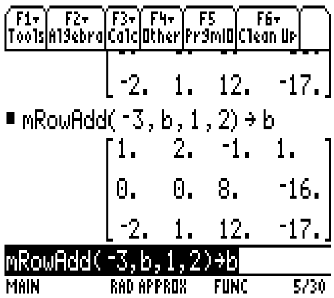

Press (Enter) and we have this screen. Notice the leading entry of row 1 is 1, just as we wanted, and our new matrix is stored in matrix B.




Now, let's turn the 3 in row 2 into 0. Multiply row 1 by −3 and add it to row 2, creating a new row 2. To do this, we need to give the command `mRowAdd(−3,B,1,2)` and store the result in B. Press (2nd) and (Math), select `4:Matrix`, hit the up arrow once and select `J:Row ops`. Now select `4:mRowAdd(` and we see the left screen. We need to enter `−3,B,1,2)` so type `−3,` (Alpha) and (B). Type the `,1,2)` to finish the expression. This says, in matrix B, multiply the first row specified (row 1) by −3, add the result to the second row specified (row 2), and place the final result back in the second row specified (row 2). Now store this answer in matrix B by using (Sto→), (Alpha) and (B), and we see the middle screen. Press (Enter) to execute the command, and we see the right screen.



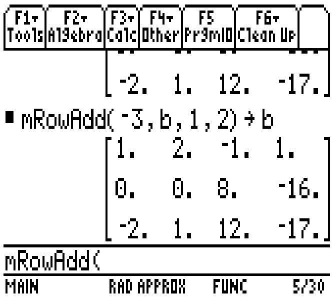


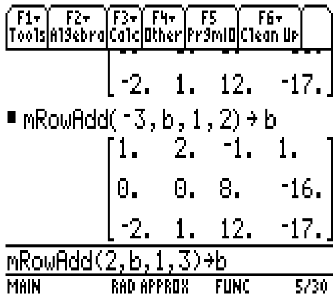


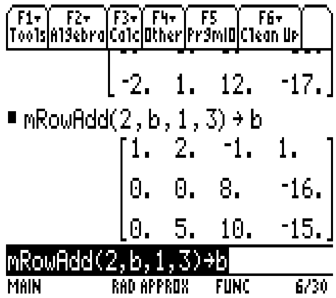
(To see our current matrix, look at the matrix in the left screen below.)

To turn the $-2$ in row 3 into 0, we need to multiply row 1 by 2, add the result to row 3 and replace row 3. We need to enter `mRowAdd(2,B,1,3)` and store the result in B.

Press [2nd] and [Math], select `4:Matrix`, hit the up arrow once and select `J:Row ops`. Now select `4:mRowAdd(` and we see the left screen. Enter `2,B,1,3)` and store the matrix back into B, to see the middle screen. After pressing [Enter], we see the right screen.

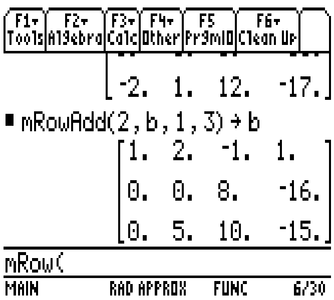


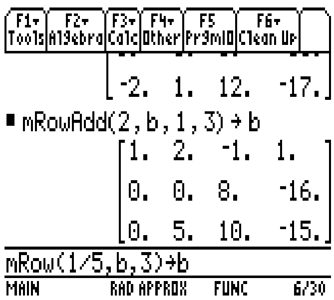


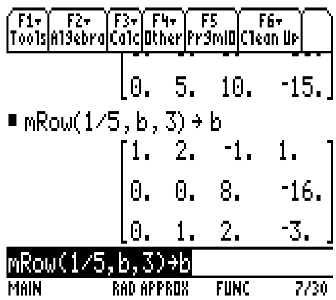



Next, multiply row 3 by $\frac{1}{5}$ to get our leading entry of 1 in row 3. We need to enter `mRow(1/5,B,3)` and store the result in B.

Directions if needed: Press [2nd] and [Math], select `4:Matrix`, hit the up arrow once and select `J:Row ops`. Now select `3:mRow(` and we see the left screen. Enter `1/5,` and press [Alpha] and [B]. Finish typing `,3)` and store it in matrix B by pressing [Sto→], then [Alpha] and [B]. We see the middle screen. Press [Enter] to see the right screen.

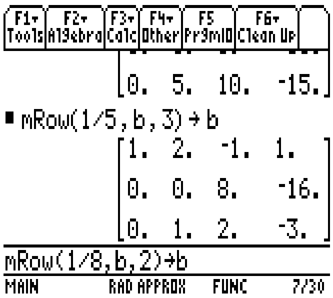


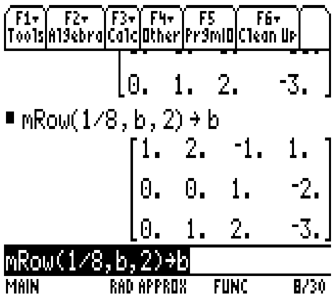


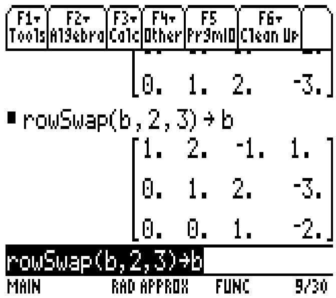



To finish placing the matrix into row-echelon form, we need to change the 8 in row 2 to a 1 by multiplying row 2 by $\frac{1}{8}$. We need to enter `mRow(1/8,B,2)`

Directions if needed: Press 2nd and Math, select `4:Matrix`, hit the up arrow once and select `J:Row ops`. Select `3:mRow(` and enter `1/8,B,2)` and store it in matrix B by pressing Sto→, then Alpha and B, and we see the left screen. Press Enter to see the middle screen. An unnecessary step we could add would be to swap rows 2 and 3. Do so by entering `rowSwap(B,2,3)`, storing it in matrix B if you wish, and we see the right screen.







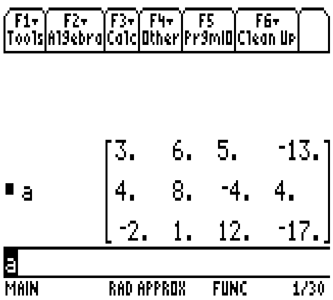

We have our row-echelon form of the matrix. The system of equations now looks like

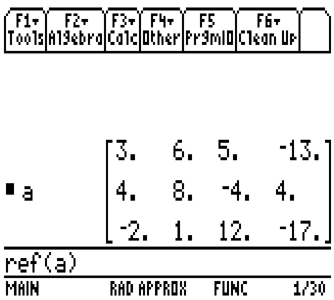
$$\begin{cases} 1x + 2y - 1z &= 1 \\ 1y + 2z &= -3 \\ 1z &= -2 \end{cases}$$

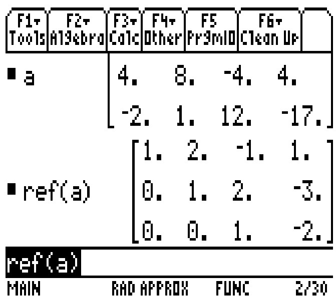
By using back substitution, we can now finish solving the system of equations.

If the instructor allows on some problems, you may go straight from the augmented matrix to the row-echelon form of the matrix by doing the following.

After you have entered the augmented matrix A (see left screen), go to the matrix options menu by pressing 2nd and Math, select `4:Matrix`, select `3:ref(` which stands for *row-echelon form*. Enter matrix A by pressing Alpha and A, and close the parentheses. We see the middle screen. Press Enter to see the final result (the right screen).

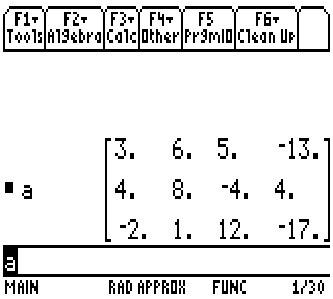


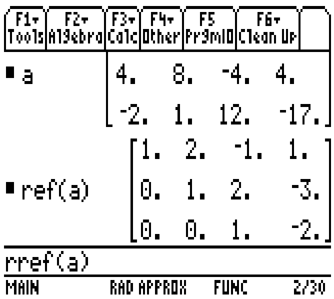


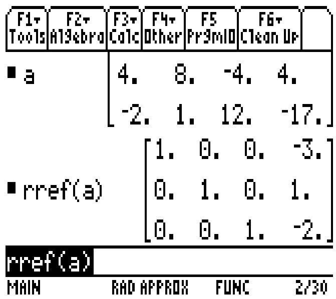


Another option we can use, if the instructor allows, is the *reduced row-echelon form* command. This takes the augmented matrix and places it into reduced row-echelon form immediately.

Again after you have entered the augmented matrix A (see left screen), go to the matrix options menu by pressing 2nd and Math, select `4:Matrix`, select `4:rref(` which stands for *reduced row-echelon form.* Enter matrix A by pressing Alpha and A and close the parentheses. There is probably no need to store this answer. So, our screen looks like the middle picture. Press Enter to see the final reduced row-echelon form of the matrix (the right screen).








From this last form, we can read our final answer: $x = -3, y = 1, z = -2.$

Press F1 and 8 to clear most of the screen, and Clear to erase the command line portion.

These directions may look very complicated upon first reading. However, after you try them a few times and get used to what needs to be done, they will flow much easier and faster. Like most things, it does take practice to become fluent with the process.

## TI-86: Matrices

We are solving this system $\begin{cases} 3x+6y+5z &= -13 \\ 4x+8y-4z &= 4 \\ -2x+y+12z &= -17 \end{cases}$

with an augmented matrix of $\begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$

### Enter the augmented matrix

Press [2nd] and [Matrx], where `Matrx` is the second function on the [7] key, to see options of `Names, Edit, Math, Ops, Cplx`. Press [F1 Names]. If you see anything in the lower row, that means you already have some matrices defined. My calculator has three matrices defined, called A, B, Mat1. Let's delete these previously defined matrixes before we continue.

If you do not need to delete any matrices, continue after the Delete section.

### Deleting matrices previously defined

To erase the matrices already defined, press [2nd] and [Mem], where `Mem` is the second function on the [3] key, to see options `Ram, Delet, Reset, Tol, ClrEnt`. Press [F2 Delet] to see options `All, Real, Cplx, List, Vectr▶`. Press [More] to see options `Matrx, Strng, Equ, Cons, Prgm▶`. We want the Matrix option, so press [F1 Matrx].

With the arrow beside the first matrix (mine is A), press [Enter] and matrix A is deleted. Delete the rest of the matrices defined.

To exit this part, press [2nd] and [Quit]

### Enter the augmented matrix continued

If needed, press [2nd] and [Matrx] again. We want to enter our matrix, so we need the `Edit` option. Press [F2 Edit] and you are asked for the name of the matrix. Suppose we name it A by pressing [A] (notice we do not press [Alpha], just the [A]). Press [Enter].

We need to specify the dimensions of the matrix, and the value of each element in the matrix. We have a $3 \times 4$ matrix to enter, 3 rows and 4 columns. Type 3 and press [Enter], and 3 rows are created. Type 4 and press [Enter], and 4 columns are created. Notice the row and column designation in the bottom left corner, `1,1=0`. This shows we are currently at row 1, column 1, and the current value of that element is zero.

Type each value from the augmented matrix, pressing Enter after each entry.

$$\begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

Notice that only 3 columns are displayed at one time in the calculator window. We can move from element to element, row to row, column to column with the arrow keys at any time. After we finish entering the numbers for the elements of the matrix, we are seeing columns 2, 3, and 4 of the matrix. Notice, the bottom left corner shows the cursor's current position, `3, 4=-17`, which is row 3, column 4, with a value of −17.

We need to exit the `Edit` mode in order to perform our matrix operations. Press 2nd and Quit.

To see matrix A that we just entered, press 2nd and Matrx, then F1 Names. Select matrix A by pressing F1. Then press Enter to see the matrix itself.

## Perform elementary row operations

We have four choices on the calculator, same as the rules we are given algebraically.

- Swap two rows, called `rSwap`.
  Example: `rSwap(A,2,3)` swaps rows 2 and 3 in matrix A.
- Add two rows, called `rAdd`.
  Example: `rAdd(A,1,2)` adds row 1 and 2, placing the result in row 2.
- Multiply one row by a nonzero constant, called `multR`.
  Example: `multR(-5,A,2)` multiples row 2 by −5, placing the result in row 2.
- Multiply one row by a nonzero constant and add that result to another row, called `mRAdd`.
  Example: `mRAdd(-5,A,3,2)` multiplies row 3 by −5, adds the result to row 2, placing the result in row 2.

None of these operations will *save* the resulting matrix. We have to save the new matrix by using the `Store` command. Continue reading for directions.

To see how to use each of these elementary row operations, we will do the appropriate steps to put the given augmented matrix into row-echelon form.

As we go through the next directions, you will notice that as each operation is performed on the matrix, the calculator screen scrolls, showing a portion of the previous matrix, the command given, and the new matrix. After a few screens, you will get used to finding the new matrix and ignoring the old work.

When performing elementary row operations, it is possible that something like `1E-13` may be used as an approximation for zero due to the way the calculator performs calculations. If this occurs, the number should be treated as zero. It is strongly suggested that you return to the Matrix Edit screen and replace the entry of `1E-13` with a 0.

Our augmented matrix is $\begin{bmatrix} 3 & 6 & 5 & -13 \\ 4 & 8 & -4 & 4 \\ -2 & 1 & 12 & -17 \end{bmatrix}$

Let's first multiply each element in row 2 by $\frac{1}{4}$, so we create a leading entry of 1 for row 2. We need to enter `multR(1/4,A,2)`.

Press [2nd] and [F4 Ops] to see options `dim, Fill, ident, ref, rref▶`.

Press [More] to see `aug, rSwap, rAdd, multR, mRAdd▶`.

We need `multR`, so press [F4 multR] to see the command appear on the screen below the matrix. Type `1/4,` and press [Alpha] and [A] to indicate matrix A. Type `,2)` to finish the command. We are saying to multiply row 2 in matrix A by $\frac{1}{4}$, and replace row 2 with the result.

Press [Enter] to see the new matrix. $\begin{bmatrix} 3 & 6 & 5 & -13 \\ 1 & 2 & -1 & 1 \\ -2 & 1 & 12 & -17 \end{bmatrix}$

However, the calculator does not save this new matrix. Since we want to do more to it, we have to save it for further use. Press [Sto→], then press [B]. Press [Enter] and we have said to store our answer in matrix B. (We want to keep matrix A unchanged in case we need it again.)

Since we have a 1 as the leading entry of row 2, let's swap row 1 and row 2.
Press [F2 rSwap]. To swap rows 1 and 2 in matrix B, we first tell what matrix to use, and then what rows to swap, so we need to say `rSwap(B,1,2)`.

Press [Alpha] and [B], then finish typing the command `,1,2)` but don't hit [Enter] yet! Let's also save our new matrix and place it back into matrix B to save a little time. Press [Sto→], press [B] (or some other name of your choosing).

Press [Enter] to see the new matrix.
Notice the leading entry of row 1 is 1, just as we wanted. $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 6 & 5 & -13 \\ -2 & 1 & 12 & -17 \end{bmatrix}$

Now, let's turn the 3 in row 2 into 0. Multiply row 1 by −3 and add the result to row 2, creating a new row 2. To do this, we need to give the command `mRAdd(−3,B,1,2)` and store the result in B.

Press [F5 mRAdd]. Type `-3,` then enter B by using [Alpha] and [B]. Type `,1,2)` to finish the expression. This says, in matrix B, multiply the first row specified (row 1) by −3, add the result to the second row specified (row 2), and place the final result back in the second row specified (row 2).

Now store this answer in matrix B by using [Sto→] and [B]. After we press [Enter] to execute the command, we see the new matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 8 & -16 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

To turn the −2 in row 3 into 0, we need to multiply row 1 by 2, add the result to row 3 and replace row 3. We need to enter `mRAdd(2,B,1,3)` and store the result in B.

Press [F5 mRAdd] and type `2,B,1,3)`.
Remember to use [Alpha] and [B].
Store the matrix back into B (don't use [Alpha] here).
After pressing [Enter], we see the new matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 8 & -16 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

Next, multiply row 3 by $\frac{1}{5}$ to get our leading entry of 1 in row 3. We need to enter `multR(1/5,B,3)` and store the result in B.

Directions if needed: Press [F4 multR] and enter `1/5,` and indicate matrix B by pressing [Alpha] and [B].

Finish typing `,3)` and store it in matrix B by pressing [Sto→] then [B]. Press [Enter] to see the new matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 8 & -16 \\ 0 & 1 & 2 & -3 \end{bmatrix}$$

To finish placing the matrix into row-echelon form, we need to change the 8 in row 2 to a 1 by multiplying row 2 by $\frac{1}{8}$. We need to enter `multR(1/8,B,2)`

Directions: Press [F4 multR], and enter `1/8,` then press [Alpha] and [B]. Finish typing `,2)` and store the result in matrix B by pressing [Sto→], then [B]. Press [Enter] to see the new matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & 2 & -3 \end{bmatrix}$$

An unnecessary step we could add would be to swap rows 2 and 3. Do so by entering `rSwap(B,2,3)`, storing the result in matrix B if you wish.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

We have our row-echelon form of the matrix. The system of equations now looks like

$$\begin{cases} 1x + 2y - 1z &= 1 \\ 1y + 2z &= -3 \\ 1z &= -2 \end{cases}$$

By using back substitution, we can now finish solving the system of equations.

If the instructor allows on some problems, you may go straight from the augmented matrix to the row-echelon form of the matrix by doing the following.

After you have entered the augmented matrix A, and have it on the screen if you wish, go to the matrix options menu by pressing [2nd] and [F4 Ops] to see the options `dim, Fill, ident, ref, rref▶`.

Press [F4 ref] which stands for *row-echelon form.* Enter matrix A (using [Alpha] and [A]). Press [Enter] to see the final result.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Another option we can use, if the instructor allows, is the *reduced row-echelon form* command. This takes the augmented matrix and places it into reduced row-echelon form immediately.

Again after you have entered the augmented matrix A, and have it on the screen if you wish, go to the matrix options menu by pressing [2nd] and [F4 Ops].

Press [F5 rref], which stands for *reduced row-echelon form.* Enter matrix A (using [Alpha] and [A]). Press [Enter] to see the final reduced row-echelon form of the matrix.

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

From this last form, we can read our final answer: $x = -3, y = 1, z = -2.$

Press [2nd] and [Quit] to exit the matrix operations, and [Clear] to erase the screen.

These directions may look very complicated upon first reading. However, after you try them a few times and get used to what needs to be done, they will flow much easier and faster. Like most things, it does take practice to become fluent with the process.