0.10 Solving Inequalities by Graphing

Solving inequalities by graphing uses techniques very similar to those we used for solving equations (see section 0.9).

0.10.1 $x-$intercept Method

Review the $x-$intercept method in section 0.9.1 if needed.

Suppose we want to solve $x^3 - 4x^2 - 5x > (x-1)^2$, which means we need to find all values of x for which this inequality is true.

We must

- Compare everything to zero: $x^3 - 4x^2 - 5x - (x-1)^2 > 0$
- Set $y =$ the left (nonzero) side.

  So, we have $y = x^3 - 4x^2 - 5x - (x-1)^2$

  and we must solve $y > 0$.
- Graph $y = x^3 - 4x^2 - 5x - (x-1)^2$.

  Determine the values of x for which $y > 0$. Thus, we need to decide where the graph is *above* the $x-$axis since that is where y is positive.

This method works in the same fashion for $<$, $\leq$, $>$, $\geq$.

If we have $<$, we include where the graph is below the $x-$axis ($y < 0$).
If we have $\leq$, we include where the graph is below the $x-$axis ($y < 0$) or on the $x-$axis ($y = 0$).

If we have $>$, we include where the graph is above the $x-$axis ($y > 0$).
If we have $\geq$, we include where the graph is above the $x-$axis ($y > 0$) or on the $x-$axis ($y = 0$).

Choose the particular directions below for your calculator. Try these instructions on your calculator as you read. If you have any difficulties, ask for help.

TI-83 (see page 148), TI-89 (see page 149), TI-86 (see page 150).

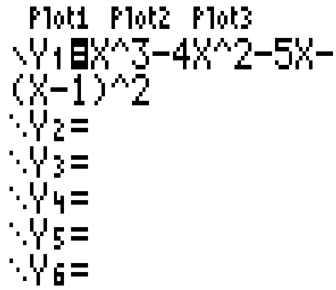
Caution: Refer to page 159 for an *unacceptable* method for solving inequalities by graphing.

TI-83: $x-$intercept Method

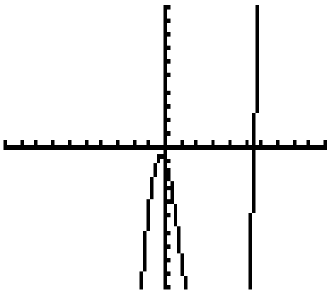
Solve $x^3 - 4x^2 - 5x > (x-1)^2$ by using $x^3 - 4x^2 - 5x - (x-1)^2 > 0$.

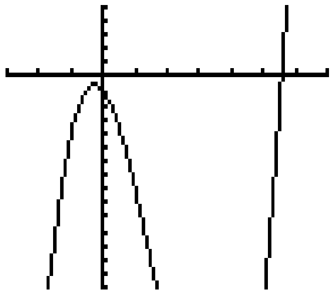
Press [Y=] and clear any function(s) left over from previous work.

Enter the formula for $y_1 = x^3 - 4x^2 - 5x - (x-1)^2$.
(You do not need to change the formula algebraically.
Type in the function as it appears.)



Using a standard window of $[-10, 10]$ by $[-10, 10]$ by pressing [Zoom] and selecting `6:ZStandard`, we get the left picture. Adjusting the window a little, we might try $[-3, 7]$ by $[-15, 5]$ to see the right picture.

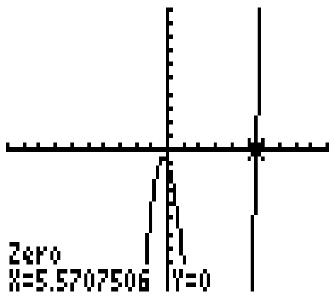




Although we are not seeing the complete graph, we recognize it to be logical for a cubic function. So, we may proceed with finding where $y > 0$. (You must be careful here about making assumptions regarding the graph and whether you are seeing enough to make your decisions. When we study polynomial functions you will have additional information to help you avoid making erroneous assumptions.)

Find the $x-$intercepts, where $y = 0$.

Looking carefully, we see only one $x-$intercept.
Using the `zero` function, starting with [2nd] and [Calc], we find $x = 5.5707506$.



Since we want all values of x for which $y > 0$, we want all values of x for which the y values are positive (the graph is above the $x-$axis).

To two decimal places, our answer for $x^3 - 4x^2 - 5x - (x-1)^2 > 0$, or $y > 0$, is $(5.57, \infty)$.
Thus, $x^3 - 4x^2 - 5x > (x-1)^2$ when x is in $(5.57, \infty)$.

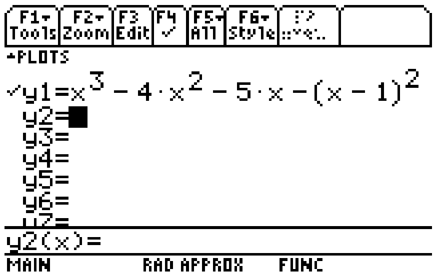
Continue reading on the bottom of page 151 for the conclusion.

TI-89: $x-$intercept Method

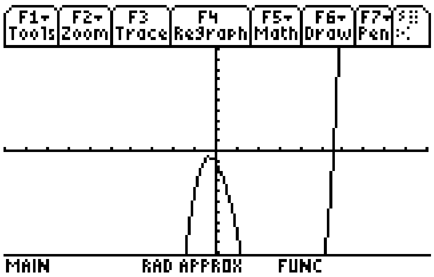
Solve $\quad x^3 - 4x^2 - 5x > (x-1)^2 \quad$ by using $\quad x^3 - 4x^2 - 5x - (x-1)^2 > 0.$

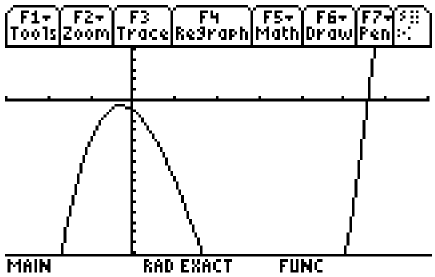
Press the green ◆ key and F1 Y= and clear any previous functions.

Enter the formula for $y_1 = x^3 - 4x^2 - 5x - (x-1)^2$.
(You do not need to change the formula algebraically.
Type in the function as it appears.)
Press Enter and verify the formula is correct



Using a standard window of $[-10, 10]$ by $[-10, 10]$ by pressing F2 Zoom and selecting `6:ZoomStd`, we get the left picture. Adjusting the window a little, we might try $[-3, 7]$ by $[-15, 5]$ to see the right picture.

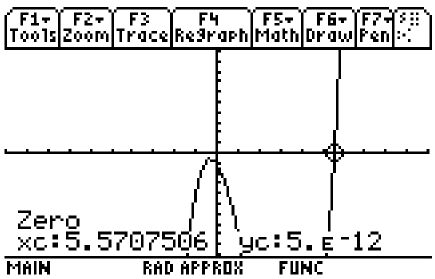




Although we are not seeing the complete graph, we recognize it to be logical for a cubic function. So, we may proceed with finding where $y > 0$. (You must be careful here about making assumptions regarding the graph and whether you are seeing enough to make your decisions. When we study polynomial functions you will have additional information to help you avoid making erroneous assumptions.)

Find the $x-$intercepts, where $y = 0$.

Looking carefully, we see only one $x-$intercept. Using the `Zero` function, starting with F5-Math, we find $x = 5.5707506$.



Since we want all values of x for which $y > 0$, we want all values of x for which the y values are positive (the graph is above the $x-$axis).

To two decimal places, our answer for $x^3 - 4x^2 - 5x - (x-1)^2 > 0$, or $y > 0$, is $(5.57, \infty)$.
Thus, $\quad x^3 - 4x^2 - 5x > (x-1)^2 \quad$ when x is in $(5.57, \infty)$.

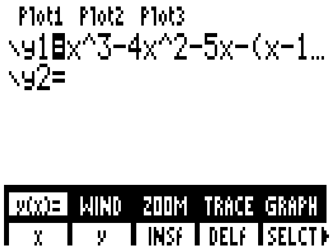
Continue reading on the bottom of page 151 for the conclusion.

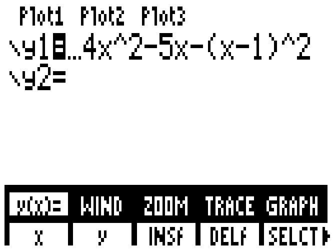
TI-86: $x-$intercept Method

Solve $x^3 - 4x^2 - 5x > (x-1)^2$ by using $x^3 - 4x^2 - 5x - (x-1)^2 > 0$.

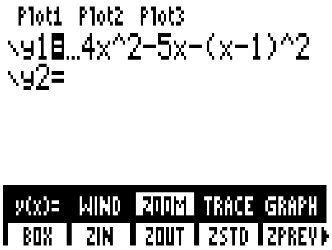
Press Graph, then F1 y(x)=, and clear any previous functions.

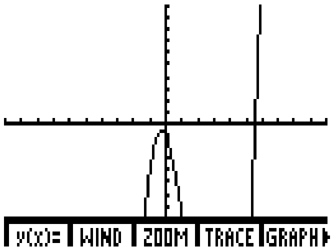
Enter the formula for $y_1 = x^3 - 4x^2 - 5x - (x-1)^2$. (You do not need to change the formula algebraically. Type in the function as it appears.) It is shown here with two pictures since the function is longer than one line.

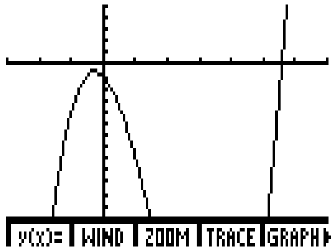




Using a standard window of $[-10, 10]$ by $[-10, 10]$ by pressing 2nd and F3 Zoom (left picture), then press F4 ZStd for `Zoom Standard`, we see the middle picture. Adjusting the window a little, we might try $[-3, 7]$ by $[-15, 5]$ to see the right picture.



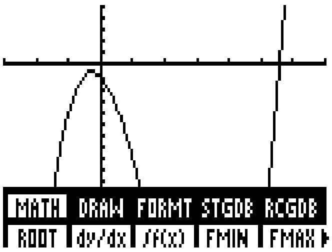


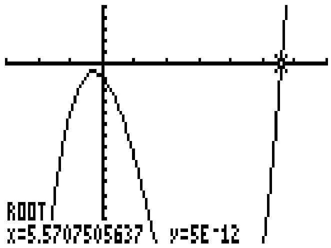


Although we are not seeing the complete graph, we recognize it to be logical for a cubic function. So, we may proceed with finding where $y > 0$. (You must be careful here about making assumptions regarding the graph and whether you are seeing enough to make your decisions. When we study polynomial functions you will have additional information to help you avoid making erroneous assumptions.)

Find the $x-$intercepts, where $y = 0$.

Looking carefully, we see only one $x-$intercept. Using More and F1 Math (left picture), then using the F1 Root, we find $x = 5.5707505637$ (right picture).





Since we want all values of x for which $y > 0$, we want all values of x for which the y values are positive (the graph is above the $x-$axis). Thus, our answer, to two decimal places, is $(5.57, \infty)$ for the inequality $y > 0$, or $x^3 - 4x^2 - 5x - (x-1)^2 > 0$. Thus,
$x^3 - 4x^2 - 5x > (x-1)^2 \quad \text{when} \quad (5.57, \infty)$.

Conclusion follows.

Conclusion: Solving Inequalities with $x-$intercepts

Assume a and b are algebraic expressions.

If we are given $a < b$, or $a > b$, or $a \leq b$, or $a \geq b$,

- Compare everything to zero, such as $a - b < 0$
  or $a - b > 0$, or $a - b \leq 0$, or $a - b \geq 0$
- Graph $y = a - b$, letting $y =$ the nonzero side of the inequality.
- Find the $x-$intercept(s).
- Determine the interval(s) for which $y < 0$; i.e., where the graph is below the $x-$axis.

  This interval (or intervals) will be the solution for $a < b$.

  If we are solving $y > 0$, then where is the graph above the $x-$axis? This interval (or intervals) will be the solution for $a > b$.

  Notice, the $x-$value is not included in the interval, since this x makes $y = 0$. Thus, with only one $x-$intercept c, the answer looks like (c, ∞) or $(-\infty, c)$.

  If we are solving, $a \leq b$, or $a \geq b$, then we must include the $x-$value in our answer, since this $x-$value makes $y = 0$ and makes $a = b$. Thus, with only one $x-$intercept c, the answer looks like $[c, \infty)$, or $(-\infty, c]$.

Continue reading on the next page for the second method for solving inequalities using points of intersection.

0.10.2 Points of Intersection Method

Review the points of intersection method for solving equations in section 0.9.2.

Suppose we want to solve $x^3 - 4x^2 - 5x > (x-1)^2$, finding all values of x for which this inequality is true.

Using the points of intersection method, we must

- Set $y_1 =$ the left side of the inequality, and
  $y_2 =$ the right side of the inequality.
  Thus, we have $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = (x-1)^2$.

- Graph $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = (x-1)^2$ on the same coordinate system.
- Determine all values of x for which $y_1 = y_2$;
  i.e., determine all values of x for which the graphs of the two functions intersect.
- To solve $x^3 - 4x^2 - 5x > (x-1)^2$, or $y_1 > y_2$, find all values of x for which the y values of y_1 are larger than the y values of y_2, or where the graph of y_1 is above the graph of y_2. Express these x values using interval notation.

Similarly, to solve $y_1 < y_2$, we find all values of x for which the graph of y_1 is below the graph of y_2.

To solve $y_1 \leq y_2$ or $y_1 \geq y_2$, we find all values of x for which the graph of y_1 is below or above the graph of y_2 appropriately, and include all values of x for which the two curves intersect.

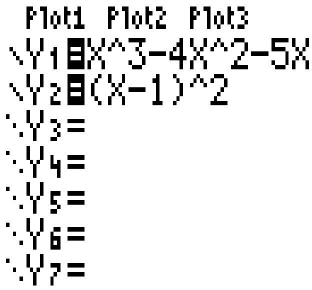
TI-83 (see page 153), TI-89 (see page 155), TI-86 (see page 157).

TI-83: Points of Intersection Method

Solve $x^3 - 4x^2 - 5x > (x-1)^2$ by setting $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = (x-1)^2$.

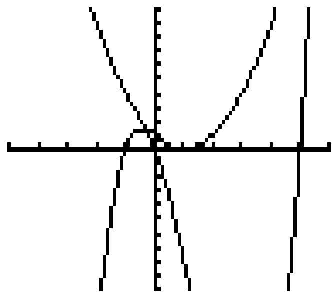
Press [Y=] and clear any function(s) left over from previous work.

Enter the formula for y_1 and y_2.



Notice the dark square on the = of y_1 and y_2. This means both functions will be graphed on the same coordinate system.

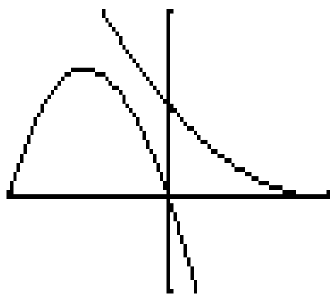
Press [Window] and enter values for $[-5, 6]$ by $[-10, 10]$. Press [Graph].



We see part of y_1, the cubic function, and we see y_2, the quadratic function (the parabola). With our current viewing window, it is hard to be sure if the graphs intersect.

Do they intersect close to $x = 0$? Change the viewing window to $[-1, 1]$ by $[-1, 2]$ to see there is no point of intersection.

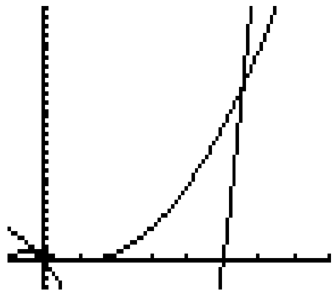
Be careful about making assumptions about what you see on the graph.



Okay, we know the two curves do not intersect near $x = 0$.

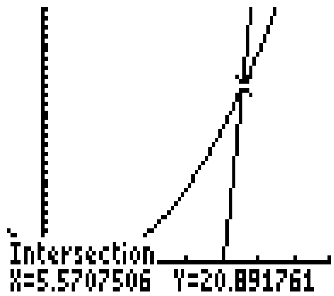
Now, do they intersect on the right side of the graph as x gets larger?

After changing the viewing window a time or two, we might try $[-1, 8]$ by $[-3, 30]$, and see this graph:



Track down the point of intersection on the right by using (2nd) and (Calc), and the `intersection` function.

If needed, refer to page 138 for more directions.



Thus, $y_1 = y_2$ when $x = 5.5707506$. We want all values of x for which $y_1 > y_2$, or when the cubic is above the parabola. Using two decimal places, we see that this happens when $x > 5.57$, the $x-$value for this point of intersection. Thus, the answer for the inequality $x^3 - 4x^2 - 5x > (x - 1)^2$ is $(5.57, \infty)$.

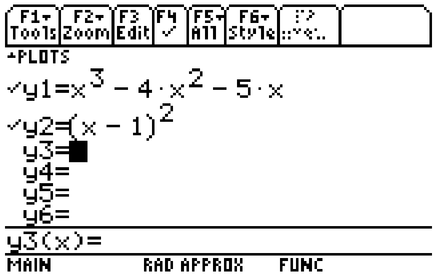
Continue reading on page 159 for a summary of this method.

TI-89: Points of Intersection Method

Solve $x^3 - 4x^2 - 5x > (x-1)^2$ by setting $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = (x-1)^2$.

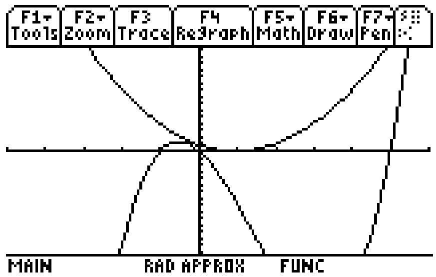
Press the green ♦ key and F1 Y=, and clear any previous functions.

Enter the formula for y_1 and y_2.
Press Enter and verify the formulas are correct.



Notice the check mark to the left of y_1 and y_2. This means both functions will be graphed on the same coordinate system.

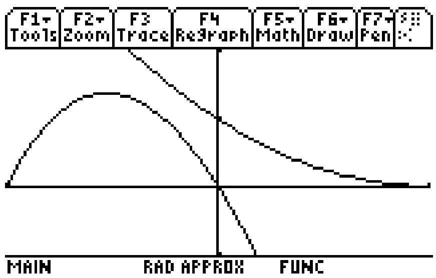
Press ♦ and F2 Window and enter values for $[-5, 6]$ by $[-15, 15]$. Press ♦ and F3 Graph.



We see part of y_1, the cubic function, and we see y_2, the quadratic function (the parabola). With our current viewing window, it is hard to be sure if the two graphs intersect.

Do they intersect close to $x = 0$? Change the viewing window to $[-1, 1]$ by $[-1, 2]$ to see there is no point of intersection.

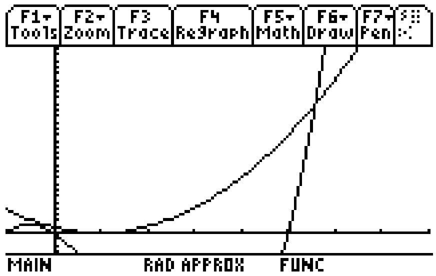
Be careful about making assumptions about what you see on the graph.



Okay, we know the two curves do not intersect near $x = 0$.

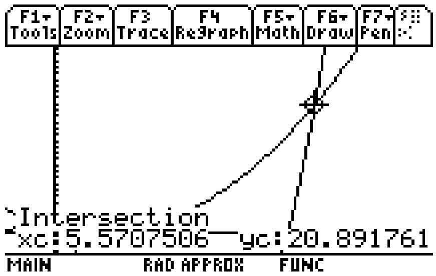
Now, do they intersect on the right side of the graph as x gets larger?

After changing the viewing window a time or two, we might try $[-1, 8]$ by $[-3, 30]$, and see this graph:



Track down the point of intersection on the right by using F5 Math and the `intersection` function.

If needed, refer to page 141 for more directions.



Thus, $y_1 = y_2$ when $x = 5.5707506$. We want all values of x for which $y_1 > y_2$, or when the cubic is above the parabola. Using two decimal places, we see that this happens when $x > 5.57$, the $x-$value for this point of intersection. Thus, the answer for the inequality $x^3 - 4x^2 - 5x > (x-1)^2$ is $(5.57, \infty)$.

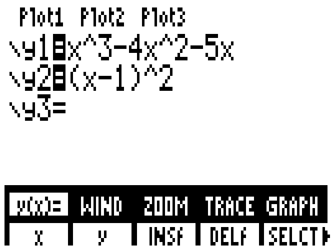
Continue reading on page 159 for a summary of this method.

TI-86: Points of Intersection Method

Solve $x^3 - 4x^2 - 5x > (x-1)^2$ by setting $y_1 = x^3 - 4x^2 - 5x$ and $y_2 = (x-1)^2$.

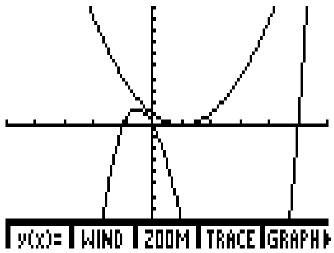
Press Graph, then F1 y(x)=, and clear any previous functions.

Enter the formula for y_1 and y_2.



Notice the dark square on the = of y_1 and y_2. This means both functions will be graphed on the same coordinate system.

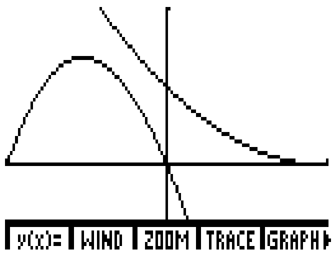
Press 2nd and F2 Wind and enter values for $[-5, 6]$ by $[-10, 10]$. Press Graph.



We see part of y_1, the cubic function, and we see y_2, the quadratic function (the parabola). With our current viewing window, it is hard to be sure if the two graphs intersect.

Do they intersect close to $x = 0$? Change the viewing window to $[-1, 1]$ by $[-1, 2]$ to see there is no point of intersection.

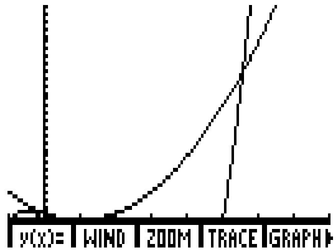
Be careful about making assumptions about what you see on the graph.



Okay, we know the two curves do not intersect near $x = 0$.

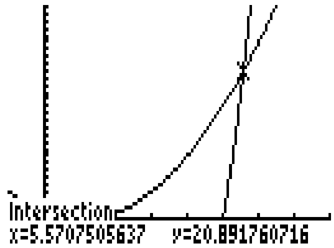
Now, do they intersect on the right side of the graph as x gets larger?

After changing the viewing window a time or two, we might try $[-1, 8]$ by $[-3, 30]$, and see this graph:



Track down the point of intersection on the right by using (More), (F1 Math) and (More) to find the `intersection` function.

If needed, refer to page 144 for more directions.



Thus, $y_1 = y_2$ when $x = 5.5707505637$. We want all values of x for which $y_1 > y_2$, or when the cubic is above the parabola. Using two decimal places, we see that this happens when $x > 5.57$, the $x-$value for this point of intersection. Thus, the answer for the inequality $x^3 - 4x^2 - 5x > (x - 1)^2$ is $(5.57, \infty)$.

Continue reading on the next page for a summary of this method.

Conclusion: Solving Inequalities with Points of Intersection

Assume a and b are algebraic expressions.

If we are given $a < b$, or $a > b$, or $a \le b$, or $a \ge b$, we must

- Graph $y_1 = a$ (the left-side of the inequality) and
  graph $y_2 = b$ (the right-side of the inequality).
- Find the $x-$coordinate of the point(s) of intersection.
  This $x-$value is the solution to the equation $y_1 = y_2$ or $a = b$.
- To solve $a < b$, determine the interval(s) for which $y_1 < y_2$; i.e. where the y_1 graph is below the y_2 graph.

  To solve $a > b$, determine the interval(s) for which $y_1 > y_2$; i.e, where the y_1 graph is above the y_2 graph.

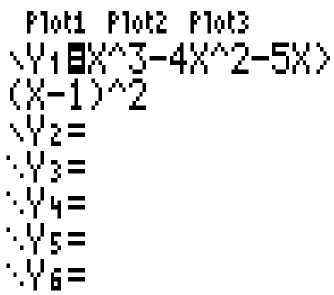
  Notice, the $x-$value is not included in these interval(s), since this x makes $y_1 = y_2$, or $a = b$. Thus, with only one point of intersection, say $x = c$, the answer looks like (c, ∞), or $(-\infty, c)$.

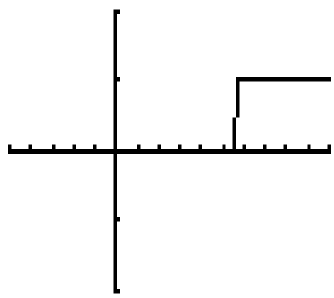
  To solve $a \le b$ or $a \ge b$, determine the interval(s) for which $y_1 < y_2$, or $y_1 > y_2$ appropriately. Then include the $x-$value in the interval, since this $x-$value makes $y_1 = y_2$, and $a = b$. Thus, with only one point of intersection, say $x = c$, the answer looks like $[c, \infty)$, or $(-\infty, c]$.

0.10.3 Caution! The following method is *unacceptable!*

The TI-83 and TI-86 will graph an inequality such as $x^3 - 4x^2 - 5x > (x-1)^2$, and show us only the solution.

Enter $y_1 = x^3 - 4x^2 - 5x > (x-1)^2$ (left picture). Using a window of $[-5, 10]$ by $[-2, 2]$ the graph looks like the right picture.





This is only a graph of the final solution $(5.57, \infty)$. This is *not* the graph we desire when we are solving the inequality by graphing. **This technique is not acceptable.** To solve an inequality by graphing, we must use either the $x-$intercept method or the points of intersection method.