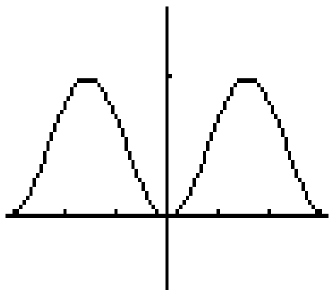
Section 7.1

1. window $[-\pi, \pi]$ by $[-.5, 1.5]$

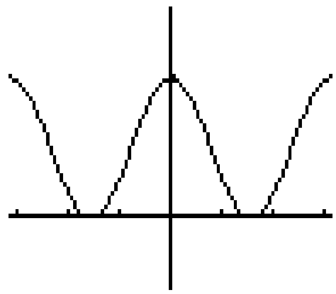
(a) $y = \sin^2(x)$

$\mathtt{y_1 = sin(x) \wedge 2}$,



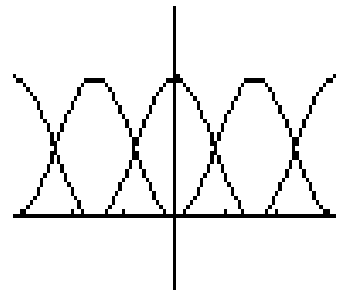
(b) $y = \cos^2(x)$

$\mathtt{y_2 = cos(x) \wedge 2}$,



(c) $y = \sin^2(x)$ and $y = \cos^2(x)$

$\mathtt{y_1 = sin(x) \wedge 2}$
$\mathtt{y_2 = cos(x) \wedge 2}$

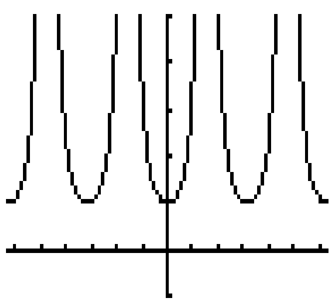


(d) Adding the $y-$values for corresponding values of x on each graph, we get a $y = 1$. For example, finding $\sin^2(1) + \cos^2(1)$ by adding the $y-$values on the graph, we get 1. (Recall function addition from chapter 2.)

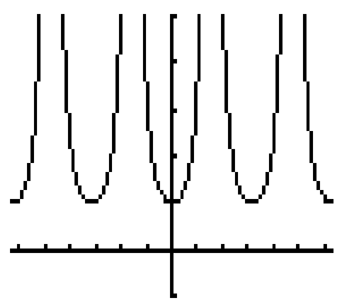
3. (a) $\tan(x)^2 + 1 = \sec^2(x)$ It is an identity.

$[-2\pi, 2\pi]$ by $[-1, 5]$ for all graphs

$\mathtt{y_1 = tan(x) \wedge 2 + 1}$

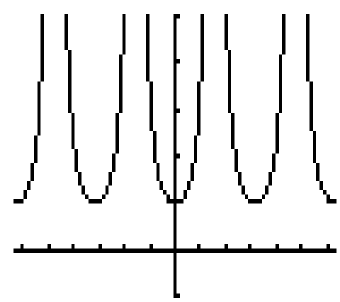


$\mathtt{y_1 = 1/cos(x) \wedge 2}$



Graphs on same system

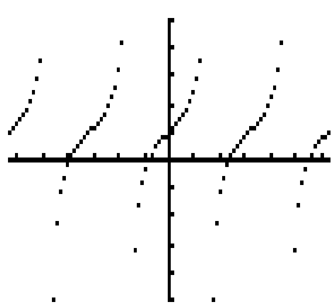
$\mathtt{y_1 = tan(x) \wedge 2 + 1}$
$\mathtt{y_2 = 1/cos(x) \wedge 2}$



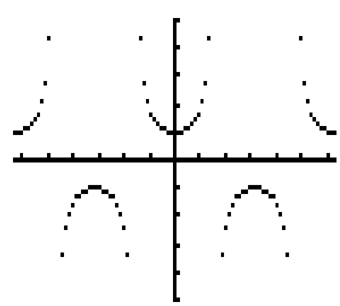
(b) $\tan(x) + 1 \neq \sec(x)$ It is not an identity.

$[-\pi, \pi]$ by $[-5, 5]$ for both graphs

$\mathtt{y_1 = tan(x) + 1}$



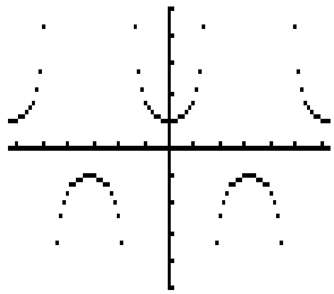
$\mathtt{y_1 = 1/cos(x)}$



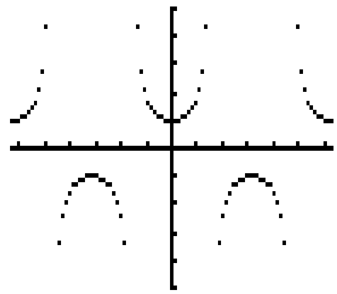
5. (a) It is an identity Show $\cos(x) + \tan(x)\sin(x) = \sec(x)$

$[-2\pi, 2\pi]$ by $[-5, 5]$ for all graphs

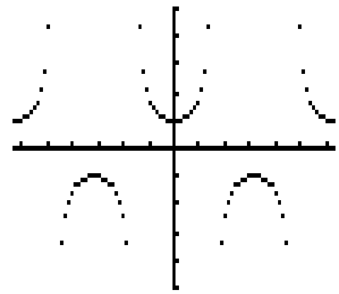
$\mathtt{y_1 = cos(x) + tan(x)sin(x)}$



$\mathtt{y_1 = 1/cos(x)}$



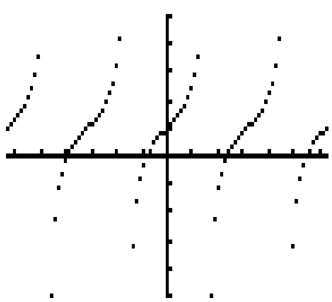
$\mathtt{y_1 = cos(x) + tan(x)sin(x)}$
$\mathtt{y_2 = 1/cos(x)}$



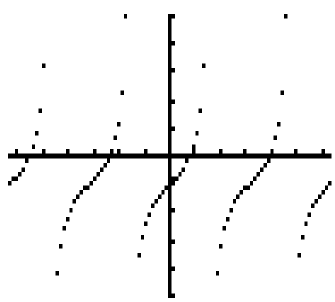
(b) It is not an identity $\tan(x) + 1 \neq \sec(x)\Big(\sin(x) - \cos(x)\Big)$

$[-2\pi, 2\pi]$ by $[-5, 5]$ for all graphs

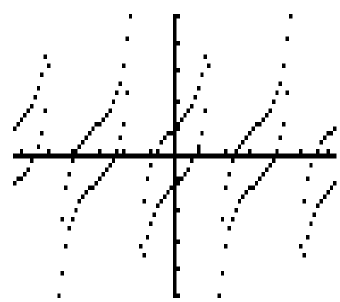
$\mathtt{y_1 = tan(x) + 1}$



$\mathtt{y_1 = sec(x)(sin(x) - cos(x))}$



$\mathtt{y_1 = tan(x) + 1}$
$\mathtt{y_2 = sec(x)(sin(x) - cos(x))}$



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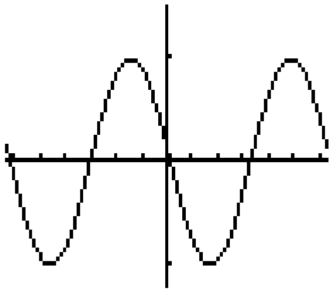
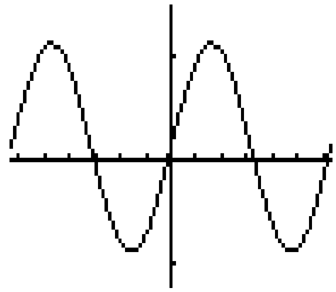
Section 7.2

1. (a) $[-2\pi, 2\pi]$ by $[-1.2, 1.5]$

$\mathtt{y_1} = \mathtt{sin}(\mathtt{x} + 3)$

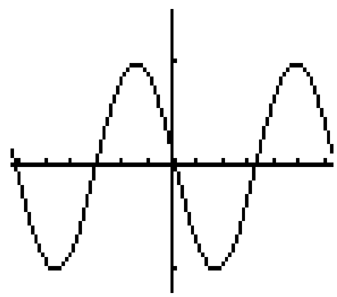


$\mathtt{y_1} = \mathtt{sin}(\mathtt{x}) + \mathtt{sin}(3)$



Tanner is correct $\quad \sin(x + 3) \neq \sin(x) + \sin(3)$.
$\sin(x + 3)$ moves the graph of $\sin(x)$ left 3 units (phase shift of -3), while $\sin(x) + \sin(3)$ shifts the graph up $\sin(3)$ units.

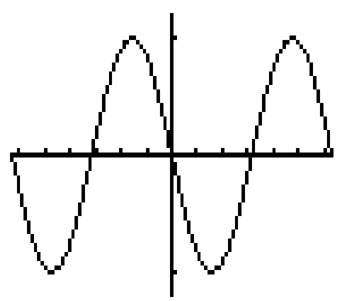
(b) $\mathtt{y_1} = \mathtt{sin}(\mathtt{x} + 3)$
$\mathtt{y_2} = \mathtt{sin}(\mathtt{x})\mathtt{cos}(3) + \mathtt{cos}(\mathtt{x})\mathtt{sin}(3)$
on the same graph, using $[-2\pi, 2\pi]$ by $[-1.5, 1.5]$.



Yes, $\sin(x + 3) = \sin(x)\cos(3) + \cos(x)\sin(3)$.

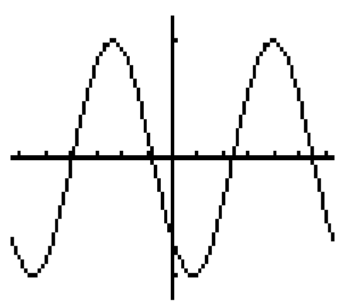
3. (a) $\sin(x - \pi) = -\sin(x)$

$\mathtt{y_1} = \mathtt{sin}(\mathtt{x} - \pi)$, $\mathtt{y_2} = -\mathtt{sin}(\mathtt{x})$
$[-2\pi, 2\pi]$ by $[-1.2, 1.2]$



(b) $\cos\left(x + \dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}\left(\cos(x) + \sin(x)\right)$

$\mathtt{y_1} = \mathtt{cos}(\mathtt{x} + 3\pi/4)$,
$\mathtt{y_2} = -\sqrt{\ }(2)/2 * (\mathtt{cos}(\mathtt{x}) + \mathtt{sin}(\mathtt{x}))$
$[-2\pi, 2\pi]$ by $[-1.2, 1.2]$



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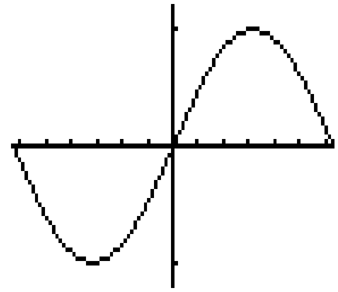
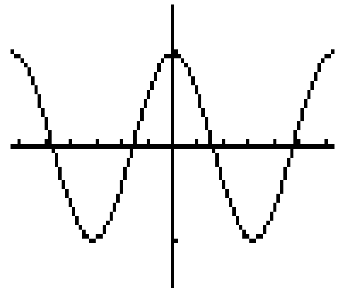
(c) $\sin(1.4x)\cos(0.9x) - \cos(1.4x)\sin(0.9x) = \sin(0.5x)$

$\mathtt{y_1 = sin(1.4x)cos(0.9x) - cos(1.4x)sin(0.9x)}$
$\mathtt{y_2 = sin(0.5x)}$
$[-2\pi, 2\pi]$ by $[-1.2, 1.2]$



5. $\mathtt{y_1 = -1/2(cos(x + \pi) + cos(x - \pi))}$
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$



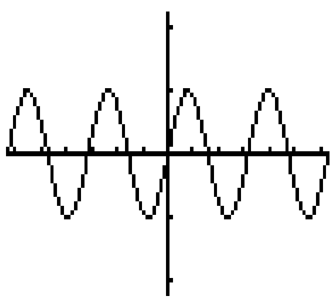
$g(x) = \cos(x)$

Show $-\frac{1}{2}\Big[\cos(x+\pi) + \cos(x-\pi)\Big] = \cos(x)$

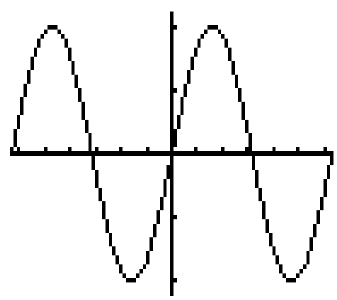
Section 7.3

1. (a) No, $\sin(2x) \neq 2\sin(x)$.

$\mathtt{y_1 = sin(2x)}$
$[-2\pi, 2\pi]$ by $[-2.2, 2.2]$



$\mathtt{y_1 = 2sin(x)}$
$[-2\pi, 2\pi]$ by $[-2.2, 2.2]$



In $\sin(2x)$, the 2 changes the period from 2π to π.
In $2\sin(x)$, the 2 changes the amplitude from 1 to 2, but does not affect the period.

Continued on next page.

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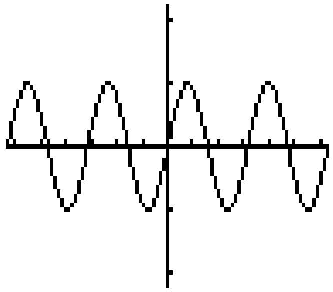
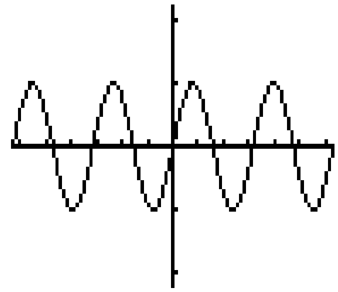
(b) Yes, $\sin(2x) = 2\sin(x)\cos(x)$.

$\texttt{y}_1 = \texttt{sin(2x)}$
$[-2\pi, 2\pi]$ by $[-2.2, 2.2]$



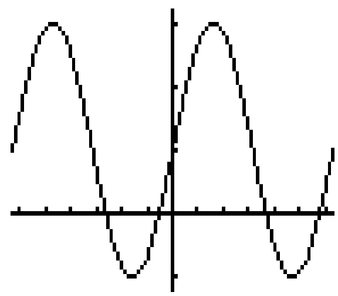
$\texttt{y}_1 = \texttt{2sin(x)cos(x)}$
$[-2\pi, 2\pi]$ by $[-2.2, 2.2]$



3. $\texttt{y}_1 = \texttt{(1 - 2cos(2x))/(2sin(x) - 1)}$
$[-2\pi, 2\pi]$ by $[-1.2, 3.2]$
period is 2π, amplitude is 2, shifted up 1

$g(x) = 2\sin(x) + 1$

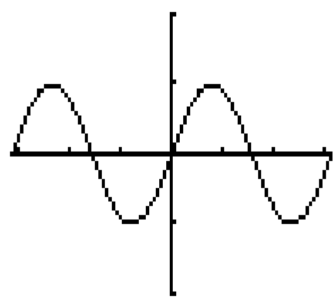
Show $\dfrac{1 - 2\cos(2x)}{2\sin(x) - 1} = 2\sin(x) + 1$



5. $\texttt{y}_1 = \texttt{2sin(x)cox(x)}$
$[-\pi, \pi]$ by $[-2, 2]$

period π, amplitude 1

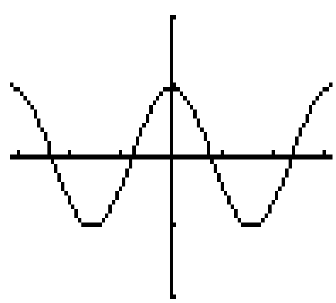
$g(x) = \sin(2x)$



7. $\texttt{y}_1 = \texttt{cos(x)} \wedge 2 - \texttt{sin(x)} \wedge 2$
$[-\pi, \pi]$ by $[-2, 2]$

period π, amplitude 1

$g(x) = \cos(2x)$



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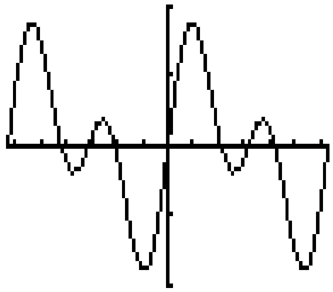
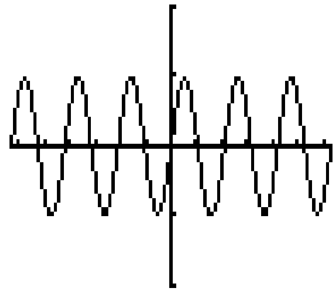
9. No, they are not equal $\sin(x) + \sin(2x) \neq \sin(3x)$

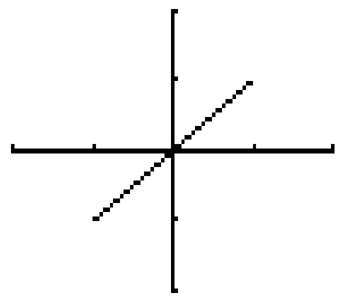
$\texttt{y}_1 = \texttt{sin(x)} + \texttt{sin(2x)}$ $\texttt{y}_1 = \texttt{sin(3x)}$

$[-2\pi, 2\pi]$ by $[-2, 2]$ for both graphs





Section 7.4

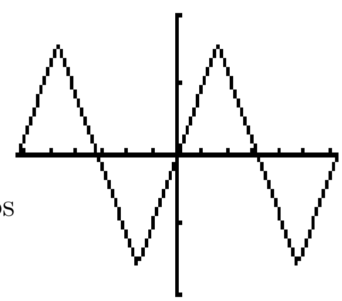


1. The function $f(x) = \sin(\sin^{-1}(x))$ only exists for a domain of $[-1, 1]$, the domain of $\sin^{-1}(x)$, but the *sine* function has a range of $[-1, 1]$ for all x values; thus, the domain of $f(x)$ is $[-1, 1]$ and the range of $f(x)$ is $[-1, 1]$, as shown in the graph.

   $\texttt{y}_1 = \texttt{sin(sin}^{-1}\texttt{(x))}$ $[-2, 2]$ by $[-2, 2]$

   Notice, this graph is a portion of the line $y = x$, or the identity function.

   For values outside of this domain $[-1, 1]$, we get no points on the graph.

   Now, to graph $g(x) = \sin^{-1}(\sin(x))$, we consider the domain of $\sin(x)$ which is $(-\infty, \infty)$. There is no reason here to restrict the domain since $\sin(x)$ maps the domain $(-\infty, \infty)$ to a range of $[-1, 1]$, which is exactly what we need for the domain of the inverse function. The inverse function then maps these values, $[-1, 1]$, to a range of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so the domain of $g(x)$ is $(-\infty, \infty)$ and the range of $g(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ as seen on the graph shown.

   $\texttt{y}_1 = \texttt{sin}^{-1}\texttt{(sin(x))}$ using $[-2\pi, 2\pi]$ by $[-2, 2]$ is shown.

(Continued on next page.)

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For a little more detail about $g(x) = \sin^{-1}(\sin(x))$:

When $\sin(x)$ produces positive answers, the inverse function maps those values to $\left(0, \frac{\pi}{2}\right]$, values in the first quadrant or on the $y-$axis.

When $\sin(x) = 0,\ \sin^{-1}(0) = 0.$

When $\sin(x)$ produces negative answers, the inverse function maps those negative values to $\left[-\frac{\pi}{2}, 0\right)$, values in the fourth quadrant or on the $y-$axis.

Thus, the range of $g(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

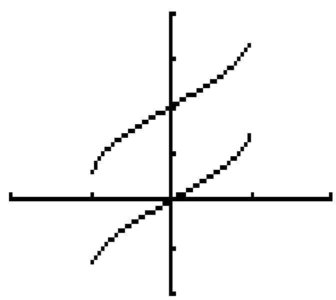
For the Inverse Function Property $\sin^{-1}(\sin(x)) = x$ to be true, $\sin(x)$ must be one-to-one for the inverse function to exist, so we must use values for x in the restricted domain of $\sin(x)$, or values of x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

When we use the full domain of $\sin(x)$, $(-\infty, \infty)$, the property is no longer true, as we see in the graph above.

3. $f(x) = \sin^{-1}(x)$ and $g(x) = \sin^{-1}(x) + 2$

   $\mathtt{y_1} = \mathtt{sin}^{-1}(\mathtt{x}),\ \mathtt{y_2} = \mathtt{sin}^{-1}(\mathtt{x}) + 2$

   $[-2, 2]$ by $[-2, 4]$

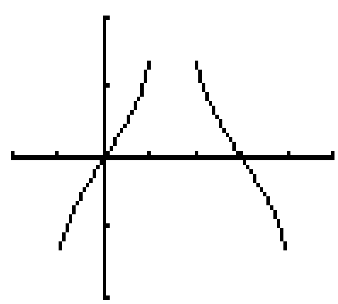


   $f(x)$ is shifted up 2 units to get $g(x)$.

5. $f(x) = \sin^{-1}(x)$ and $g(x) = -\sin^{-1}(x - 3)$

   $\mathtt{y_1} = \mathtt{sin}^{-1}(\mathtt{x}),\ \mathtt{y_2} = -\mathtt{sin}^{-1}(\mathtt{x} - 3)$

   $[-2, 5]$ by $[-2, 2]$



   $f(x)$ is reflected over the $x-$axis and shifted right 3 units to get $g(x)$.

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Section 7.5

1. When evaluating $\sin^{-1}(0.4567)$, we are looking for the real number between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose *sine* is 0.4567. In this case, the answer is between 0 and $\frac{\pi}{2}$ since 0.4567 is positive. There is only one number that satisfies this requirement.

   When solving $\sin(x) = 0.4567$, we are looking for all values of x whose *sine* is 0.4567. There are an infinite number of answers, all of which are between 0 and π, or are coterminal with an angle between 0 and π .

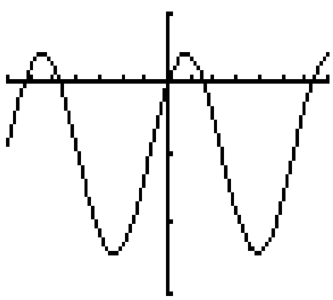
3. solutions $x = -6.2832,\ -4.7124,\ 0,\ 1.5708,\ 6.2832$

   Find at least 4 solutions to $\sin(x) + \cos(x) = \sin^2(x) + \cos^2(x)$, or using the substitution $\sin^2(x) + \cos^2(x) = 1$, find solutions to $\sin(x) + \cos(x) = 1$.

   $x-$intercepts method

   $y_1 = \sin(x) + \cos(x) - 1$

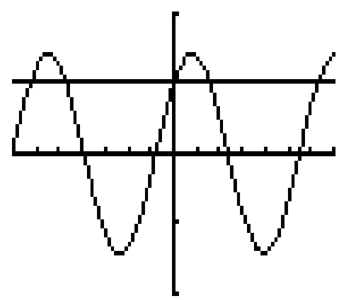
   $[-7, 7]$ by $[-3, 1]$



   points of intersection method

   $y_1 = \sin(x) + \cos(x), \quad y_2 = 1$

   $[-7, 7]$ by $[-2, 2]$



   $x-$intercepts, or the $x-$value of the points of intersection:

   $-6.283185,\ -4.712389,\ 0,\ 1.5707963,\ 6.2831853$

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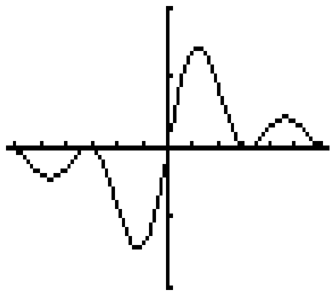
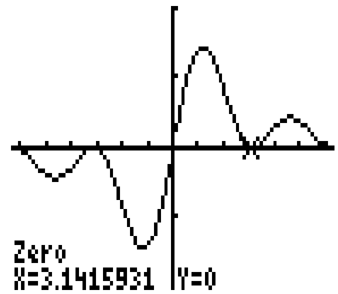
5. $f(x) = \dfrac{1-\cos(2x)}{x}$

(a) Solve $\dfrac{1-\cos(2x)}{x} = 0$ $\qquad$ $x-$intercepts: $\quad n\pi$, for n an integer

(b) $\texttt{y}_1 = (1 - \texttt{cos}(2\texttt{x}))/\texttt{x}$

$[-2\pi, 2\pi]$ by $[-2, 2]$





The $x-$intercept shown is 3.14159 which is π.

(c) odd function

(d) Show $f(-x) = -\dfrac{1-\cos(2x)}{x}$

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