Section 4.1

1. (a) transformations

   $g(x)$: right 1

   $h(x)$: reflect y, up 2

   $k(x)$: reflect x, down 1

   (b) formulas:

   $g(x) = 3^{x-1}$

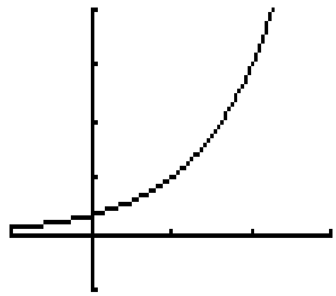
   $h(x) = 3^{-x} + 2$

   $k(x) = -3^x - 1, \quad$ or $k(x) = -1(3^x) - 1$

   (c) Be sure to draw the appropriate horizontal asymptotes.

   (d) $g(x) = 3^{x-1}$ $\qquad$ `y₁ = 3 ∧ (x − 1)`

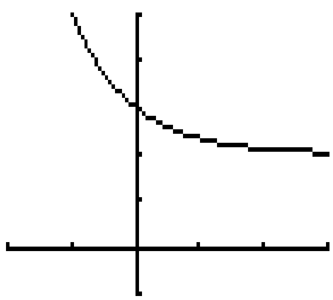
   $[-1, 3]$ by $[-1, 4]$



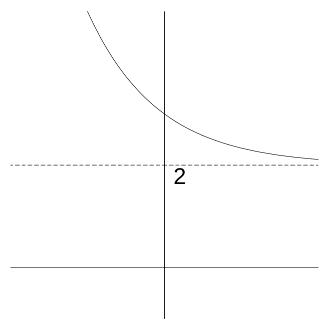
$h(x) = 3^{-x} + 2$ $\qquad$ `y₁ = 3 ∧ (−x) + 2`

$[-2, 3]$ by $[-1, 5]$

graph on calculator



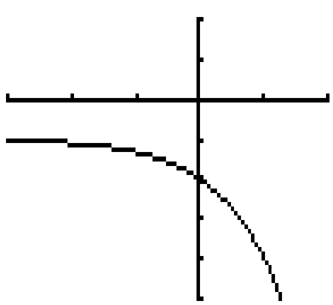
graph with asymptote



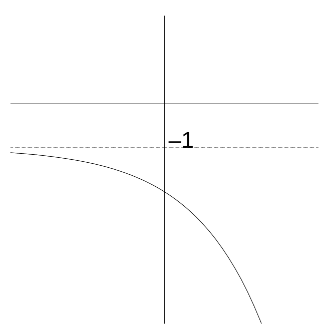
$k(x) = -3^x - 1$ $\qquad$ `y₁ = −3 ∧ x − 1`

$[-3, 2]$ by $[-5, 2]$

graph on calculator



graph with asymptote

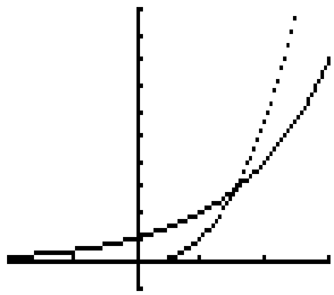


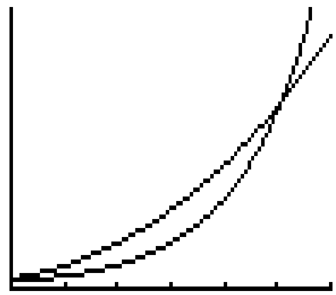
3. (a) $f(x) = 2^x$ grows faster as x gets very large

$\mathtt{y_1 = 2 \wedge x}$ $\mathtt{y_2 = x \wedge 2.5}$ shown in dot mode

$[-2, 3]$ by $[-1, 10]$, left graph shows $y_2 = x^{2.5}$ above $y_1 = 2^x$.

$[2, 8]$ by $[-1, 200]$, right graph shows the curves crossing a second time and $y_1 = 2^x$ is above $y_2 = x^{2.5}$



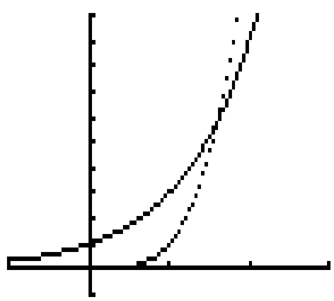


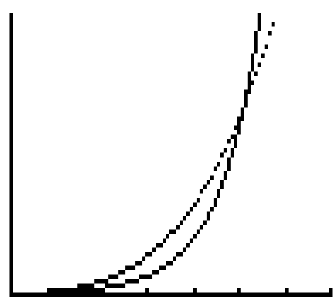
(b) $f(x) = 3^x$ grows faster as x gets very large

$\mathtt{y_1 = 3 \wedge x}$ $\mathtt{y_2 = x \wedge 3.7}$ shown in dot mode

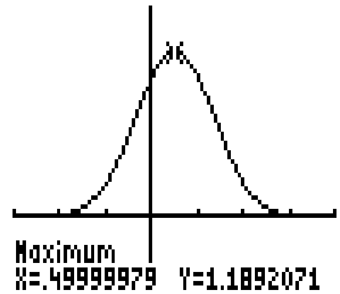
$[-1, 3]$ by $[-1, 10]$, left graph shows $y_2 = x^{3.7}$ above $y_1 = 3^x$.

$[1, 8]$ by $[-5, 1200]$, right graph shows the curves crossing a second time and $y_1 = 3^x$ is above $y_2 = x^{3.7}$.





5. $y_1 = 2 \wedge (x - x \wedge 2)$ $[-3, 4]$ by $[-0.5, 1.5]$

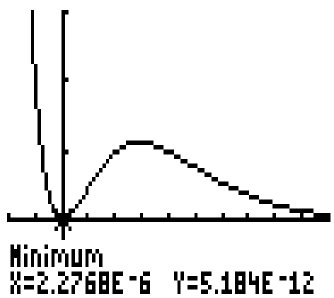


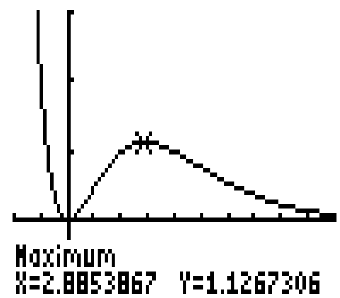
increasing: $(-\infty, .5]$ decreasing: $[.5, \infty)$

$y = 1.18920071$

maximum value is 1.19

7. $y_1 = x \wedge 2(2 \wedge (-x))$ $[-2, 10]$ by $[-1, 3]$





increasing: $[0, 2.89]$

decreasing: $(-\infty, 0] \cup [2.89, \infty)$

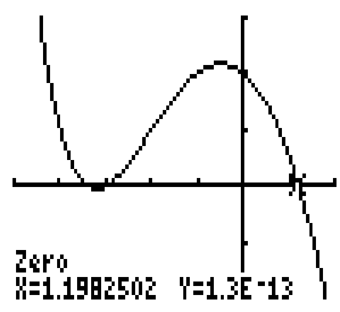
$y = 0$ (Recall, we represent $5.184\text{E} - 12$ as zero.)

minimum value is 0

$y = 1.1267306$

maximum value is 1.13

9. $y_1 = 1 - x \wedge 2 + 2 \wedge (-x)$ $[-5, 2]$ by $[-2, 3]$

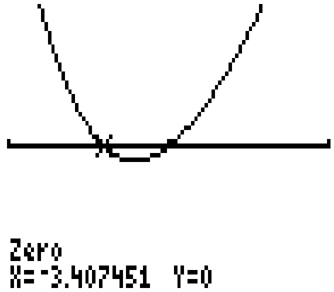


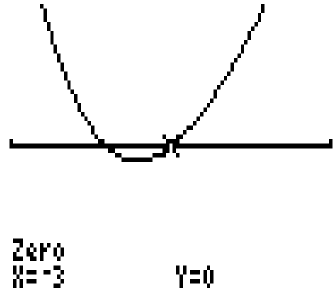
We need to find out what happens between $x = -4$ and $x = -2$.

Since we are looking for the $x-$intercepts, we can consider something like $[-.5, .5]$ for the $y-$values to show what happens on the $x-$axis.

(Continued on next page.)

So, using $[-4,-2]$ by $[-.5,.5]$, we have



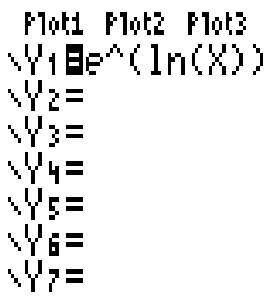


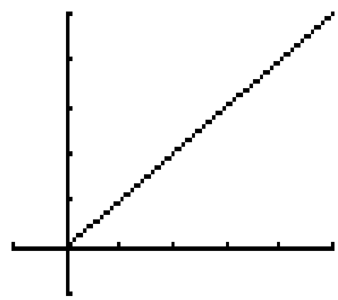
The real zeros are: $-3.407451,\ -3.00,\ 1.1982502$
To two decimal places, we have $x=-3.41,\ -3$, and 1.20.

11. $4e^x+e^{-x}+1$ graph $y_1 = e\wedge(-x)(e\wedge(x)+2)-(2e\wedge(x)+1)(e\wedge(-x)-2)$
and $y_2 = 4e\wedge(x)+e\wedge(-x)+1$ on $[-3,2]$ by $[-1,15]$

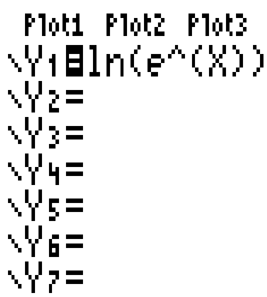
Section 4.2

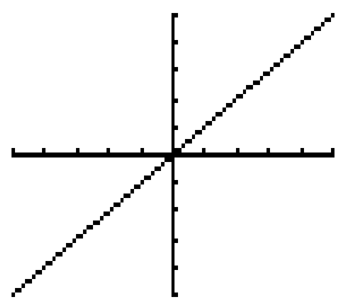
1. (a) domain $(0,\infty)$
$[-1,5]$ by $[-1,5]$





(b) domain $(-\infty,\infty)$
$[-5,5]$ by $[-5,5]$





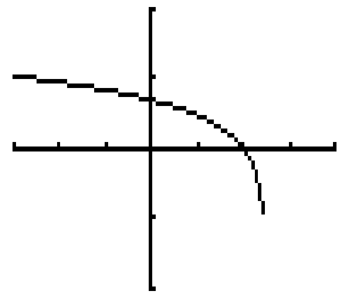
(c) $y=e^{\ln x}=x$, but the domain is $x>0$ since the domain of $\ln x$ is $x>0$. Thus, the graph of $y=x$ only exists in quadrant I.

$y=\ln(e^x)=x$, but the domain is $(-\infty,\infty)$ since the domain of $y=e^x$ is $(-\infty,\infty)$ and e^x yields only positive values for the natural logarithm to work with. Thus, the graph of $y=x$ exists for all real numbers x.

3. (a) Domain $\left(-\infty, \frac{5}{2}\right)$

$\mathtt{y_1} = \mathtt{log(5-2x)}$ $\quad [-3, 4]$ by $[-2, 2]$

vertical asymptote is $x = \frac{5}{2}$



3a. On paper:

$f(x) = \log(5-2x)$

$5 - 2x > 0$

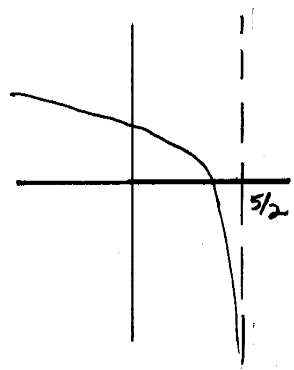
$-2x > -5$

$x < \frac{5}{2}$

Domain $\left(-\infty, \frac{5}{2}\right)$

$y_1 = \log(5-2x)$

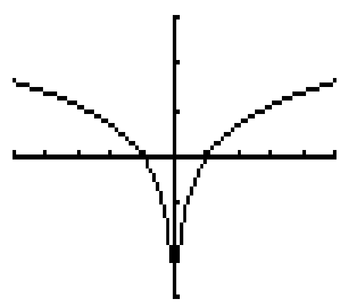
$[-3, 4]$ by $[-2, 2]$



(b) Domain $(-\infty, 0) \cup (0, \infty)$

$\mathtt{y_1} = \mathtt{ln(abs(x))}$ $\quad [-5, 5]$ by $[-3, 3]$

vertical asymptote is $x = 0$

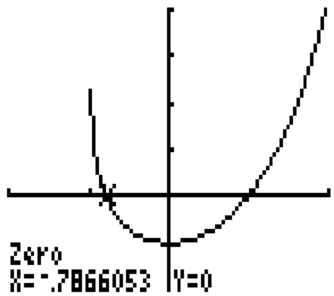


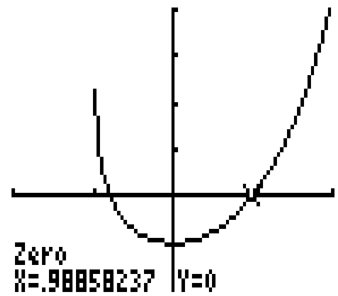
5. (a) $f(x) = g(x)$ for $x = -.79,\ .99$ (b) $f(x) < g(x)$ for $(-.79, .99)$

$f(x) = y_1 = e^x - 2$ and $g(x) = y_2 = \ln(x+1)$

<u>$x-$intercept method</u> Need $y_1 - y_2 = 0$ and $y_1 - y_2 < 0$

$\mathtt{y_1 = e \wedge (x) - 2 - ln(x+1)}$ $[-2, 2]$ by $[-2, 4]$



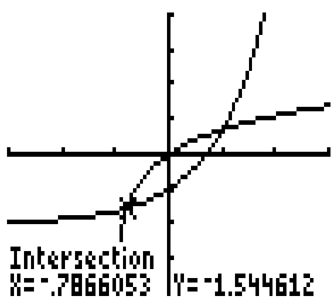


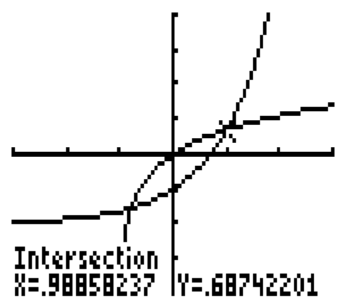
$x-$intercepts are $x = -.7866053$ and $x = .98858237$

$y_1 - y_2 < 0$ when x is between $-.7866053$ and $.98858237$

<u>points of intersection method</u> Need $y_1 = y_2$ and $y_1 < y_2$

$\mathtt{y_1 = e \wedge (x) - 2,\ y_2 = ln(x+1)}$ $[-3, 3]$ by $[-4, 4]$





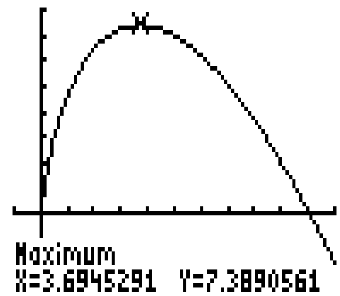
graphs intersect at $x = -.7866053$ and $x = .98858237$
$y_1 < y_2$ when x is between $-.7866053$ and $.98858237$

7. (a) $\texttt{y}_1 = \texttt{6x} - \texttt{2xln(2x)}$
$[-1, 11]$ by $[-3, 8]$

$y = 7.3890561$, maximum value is 7.39

$x = 3.6945291$

increasing $(0, 3.69]$ decreasing $[3.69, \infty)$

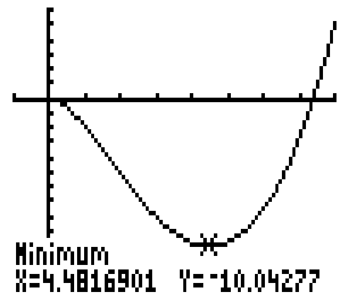


(b) $\texttt{y}_1 = \texttt{x} \wedge \texttt{2(ln(x)} - \texttt{2)}$
$[-1, 8]$ by $[-11, 6]$

$y = -10.04277$, minimum value is -10.04

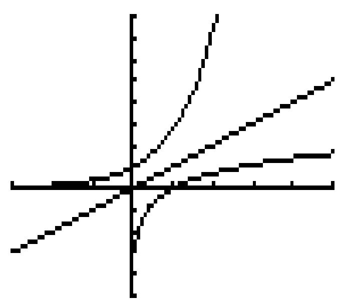
$x = 4.4816901$,

increasing $[4.48, \infty)$ decreasing $(0, 4.48]$



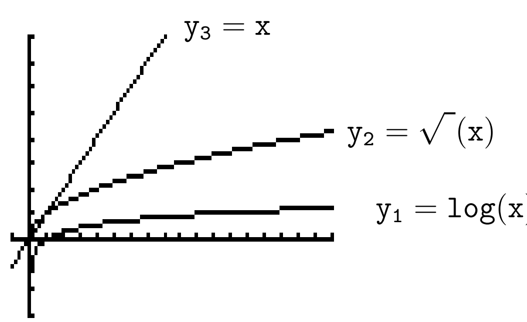
9. $g(x)$ is the reflection of $f(x)$ about the line $y = x$; thus, $f(x)$ and $g(x)$ are inverses of each other.

$\texttt{y}_1 = \texttt{e} \wedge \texttt{x}$, $\texttt{y}_2 = \texttt{ln(x)}$
$[-3, 5]$ by $[-5, 8]$



11. $f(x) = y_1$, $g(x) = y_2$, $h(x) = y_3$

$\texttt{y}_1 = \texttt{log(x)}$, $\texttt{y}_2 = \sqrt{}\texttt{(x)}$, $\texttt{y}_3 = \texttt{x}$
$[-1, 18]$ by $[-3, 8]$



So, $h(x)$, $g(x)$, $f(x)$ in order largest to smallest.

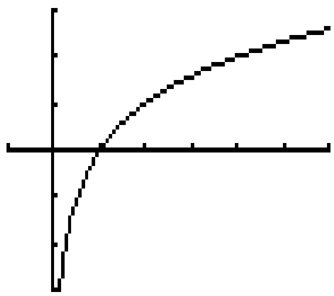
Section 4.3

1. $\log\left(\frac{x}{2}\right) = \log(x) - \log(2)$

$[-1, 6]$ by $[-3, 3]$

`y₁ = log(x/2)`

`y₁ = log(x)/log(2)`



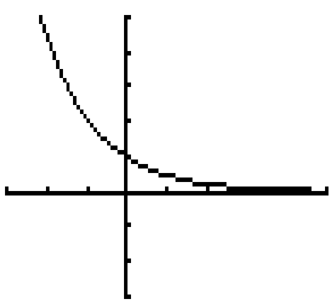
`y₁ = log(x) − log(2)`

3. They are not the same, so $(\log(3))^x \neq x\log(3)$

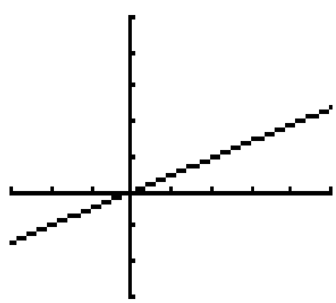
$[-3, 5]$ by $[-3, 5]$

`y₁ = (log(3)) ∧ x` is interpreted the same way as `y₁ = log(3) ∧ x` on the calculator. The exponent applies to the entire quantity. If we want `log(3^x)`, we must place the power inside the parentheses like this `y₁ = log(3 ∧ x)` .

`y₁ = log(3) ∧ x`



`y₁ = xlog(3)`



The x in $(\log(3))^x$ applies to the whole parentheses - the whole logarithm.

$\log(3)$ is a constant, so $y = x\log(3)$ is x times a constant. Thus, we have a straight line.

Section 4.4

1. solution $(2.6420, \infty)$

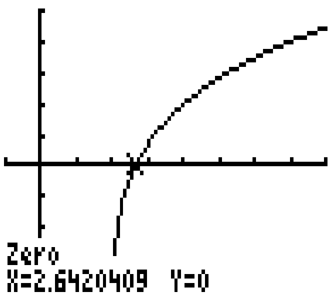
$[-1, 8]$ by $[-4, 5]$

x–intercept method

Need $\ln(x-2) + \ln(x^2) - 1.5 = 0$

then $\ln(x-2) + \ln(x^2) - 1.5 > 0$

$\texttt{y}_1 = \texttt{ln(x} - \texttt{2)} + \texttt{ln(x} \wedge \texttt{2)} - \texttt{1.5}$

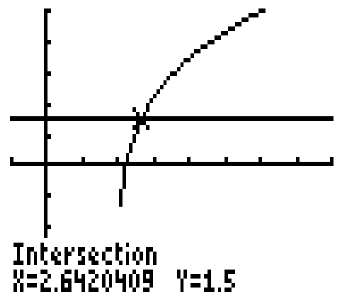


points of intersection method

Need $\ln(x-2) + \ln(x^2) = 1.5$

then $\ln(x-2) + \ln(x^2) > 1.5$

$\texttt{y}_1 = \texttt{ln(x} - \texttt{2)} + \texttt{ln(x} \wedge \texttt{2)},\ \texttt{y}_2 = \texttt{1.5}$



The x–intercept, or the x–value of the point of intersection, is 2.6420409.

Thus, we want all $x > 2.6420409$.

1. On paper:

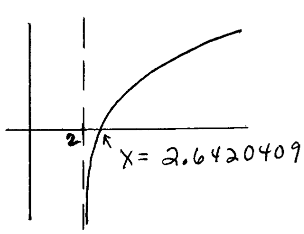
$\ln(x-2) + \ln(x^2) > 1.5$

x-intercept method

$\ln(x-2) + \ln(x^2) - 1.5 > 0$

$y_1 = \ln(x-2) + \ln(x \wedge 2) - 1.5$

$[-1, 8]$ by $[-4, 5]$



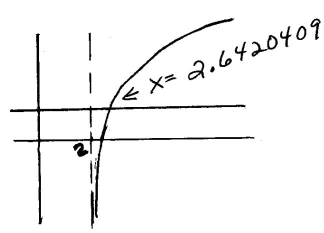
$y_1 > 0$ for $(2.6420, \infty)$

points of intersection method

$y_1 = \ln(x-2) + \ln(x \wedge 2)$

$y_2 = 1.5$

$[-1, 8]$ by $[-4, 5]$



$y_1 > y_2$ for $(2.6420, \infty)$

Solution $(2.6420, \infty)$

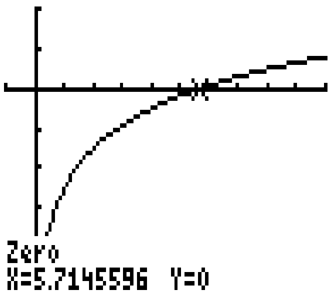
3. solution $x = 5.7146$

<u>$x-$intercept method</u>

Need $\ln(x) + \log(x) - 2.5 = 0$

$\mathtt{y_1} = \mathtt{ln(x)} + \mathtt{log(x)} - 2.5$

$[-1, 10]$ by $[-5, 2]$

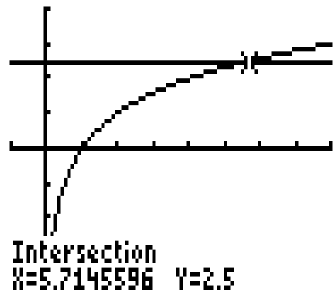


<u>points of intersection method</u>

Need $\ln(x) + \log(x) = 2.5$

$\mathtt{y_1} = \mathtt{ln(x)} + \mathtt{log(x)}$, $\mathtt{y_2} = 2.5$

$[-1, 8]$ by $[-4, 4]$

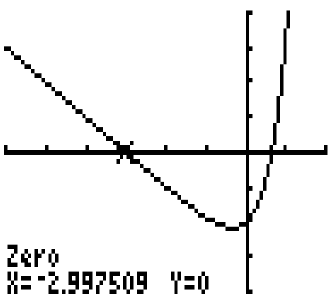


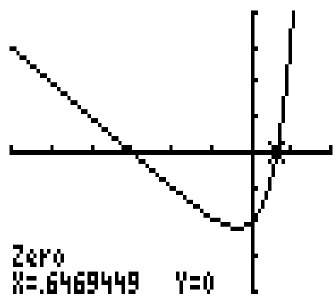
The $x-$intercept, or the $x-$value for the point of intersection, is 5.7145596.

5. solutions $x = -2.9975,\ .6469$

<u>$x-$intercept method</u>

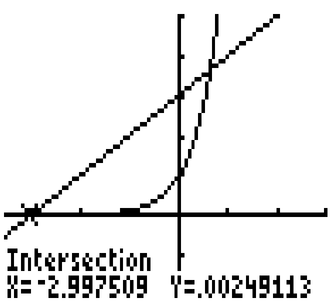
Need $e^{2x} - x - 3 = 0$ $\mathtt{y_1} = \mathtt{e} \wedge (\mathtt{2x}) - \mathtt{x} - 3$ $[-6, 2]$ by $[-4, 4]$

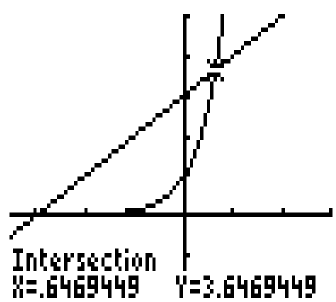




<u>points of intersection method</u>

Need $e^{2x} = x + 3$ $\mathtt{y_1} = \mathtt{e} \wedge (\mathtt{2x})$, $\mathtt{y_2} = \mathtt{x} + 3$ $[-3.5, 3]$ by $[-2, 5]$





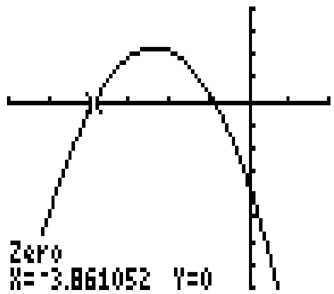
The $x-$intercepts, or $x-$values for the points of intersection, are -2.997509 and .6469449.

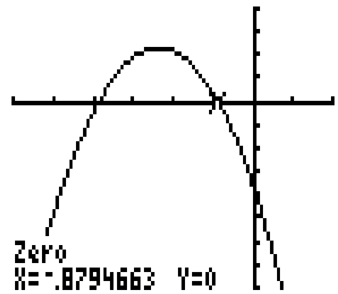
7. solutions $x = -3.8611, -.8795$

<u>$x-$intercept method</u>

Need $\ln(1.4 - x) - (x + 2.23)^2 + 1 = 0$

$\mathtt{y_1} = \mathtt{ln(1.4 - x) - (x + 2.23) \wedge 2 + 1}$ $[-6, 2]$ by $[-8, 4]$



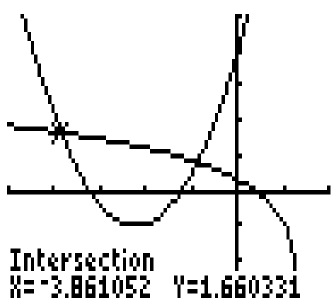


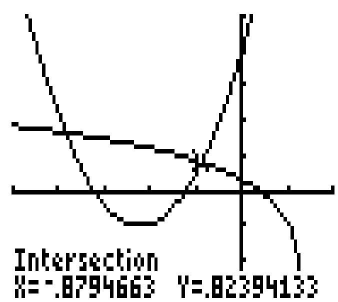
The $x-$intercepts are -3.861052 and $-.8794663$.

<u>points of intersection method</u>

Need $\ln(1.4 - x) = (x + 2.23)^2 - 1$

$\mathtt{y_1} = \mathtt{ln(1.4 - x)}$, $\mathtt{y_2} = \mathtt{(x + 2.23) \wedge 2 - 1}$ $[-5, 2]$ by $[-3, 5]$





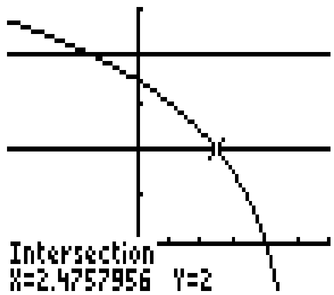
The $x-$values for the points of intersection are -3.861052 and $-.8794663$.

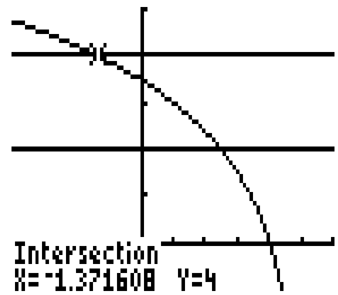
9. solution $[-1.3716, 2.4758]$

Solve $2 \le 2.16 \ln(5 - x) \le 4$

$\mathtt{y_1} = 2$, $\mathtt{y_2} = \mathtt{2.16ln(5 - x)}$, $\mathtt{y_3} = 4$ $[-4, 6]$ by $[-1, 5]$

Need all x for which $y_1 \le y_2 \le y_3$





The $x-$values for which the graph of y_2 is between y_1 and y_3 are $[-1.371608, 2.4757956]$,