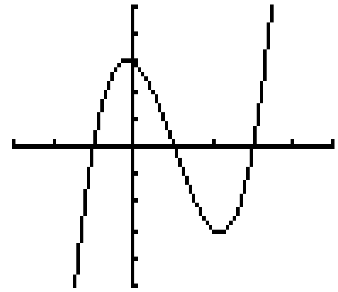
Section 3.1

1. (a) $y_1 = (x+1)(x-1)(x-3)$, $[-3, 5]$ by $[-5, 5]$

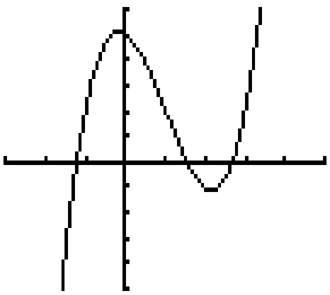


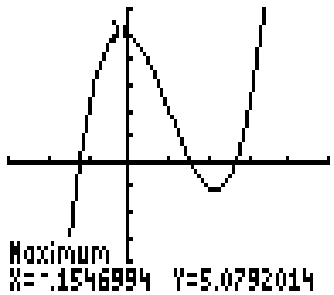
local maximum $y = 3.0792014$, so maximum value is 3.1,
local minimum $y = -3.079201$, so minimum value is -3.1.

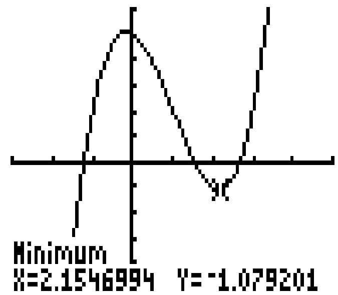
(b) $g(x)$ shifts the graph of $f(x)$ up 2 units, so we have

local maximum of $3.1 + 2$, so maximum value of $g(x)$ is 5.1,
local minimum of $-3.1 + 2$, so minimum value of $g(x)$ is -1.1.

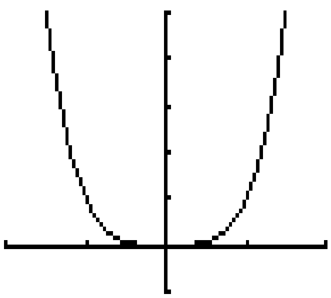
(c) $y_1 = (x+1)(x-1)(x-3) + 2$, $[-3, 5]$ by $[-5, 6]$



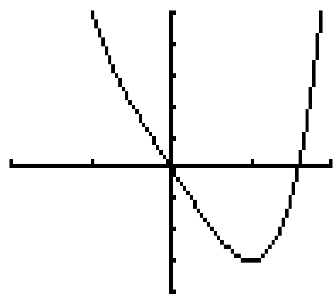




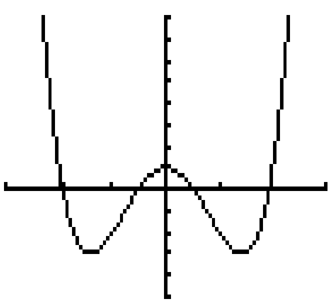
3. $y_1 = x \wedge 4$
$[-2, 2]$ by $[-1, 5]$



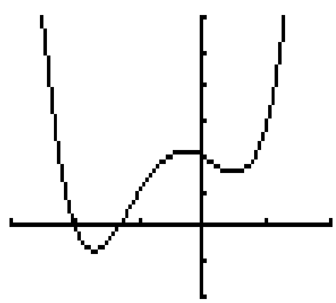
$y_2 = x \wedge 4 - 4x$
$[-2, 2]$ by $[-4, 5]$



$y_3 = x \wedge 4 - 4x \wedge 2 + 1$
$[-3, 3]$ by $[-5, 8]$



$y_4 = x \wedge 4 + 2x \wedge 3 - x \wedge 2 - x + 2$
$[-3, 2]$ by $[-2, 6]$



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$y_1 = x^4$ and $y_2 = x^4 - 4x$ have 1 turning point.

$y_3 = x^4 - 4x^2 + 1,\ y_4 = x^4 + 2x^3 - x^2 - x + 2$ have 3 turning points.

A degree 4 has 1 or 3 turning points. So, a degree 4 has at most 3 turning points.

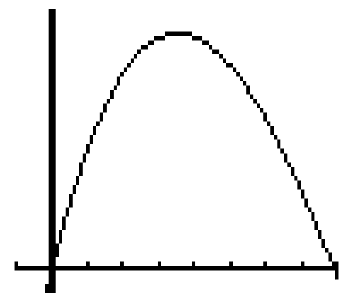
$y_1 = x^4$ has 1 $x-$intercept.

$y_2 = x^4 - 4x$ and $y_4 = x^4 + 2x^3 - x^2 - x + 2$ have 2 $x-$intercepts.

$y_3 = x^4 - 4x^2 + 1$ has 4 $x-$intercepts.

Degree 4 functions could have 0, 1, 2, 3, or 4 $x-$intercepts, but no more than 4.

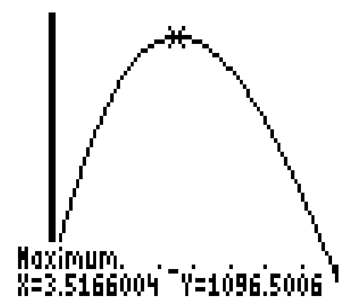
5. (a) $V(x) = x(15.8 - 2x)(42.6 - 2x)$ domain: $(0, 7.9)$



(b) $\mathtt{y_1 = x(15.8 - 2x)(42.6 - 2x)}$
$[-1, 8]$ by $[-100, 1200]$

(c) maximum volume is 1096.5 cubic inches

$y = 1096.5006$



(d) $x = 3.5165996$ square cutout is 3.5 inches by 3.5 inches

(e) Volume is larger than 1000 cubic inches for x in $(2.4, 4.7)$.

Need to solve $x(15.8 - 2x)(42.6 - 2x) > 1000$

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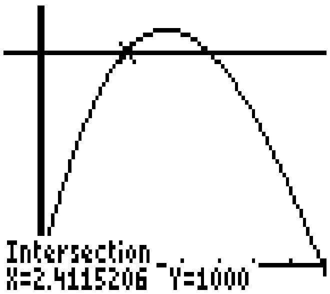
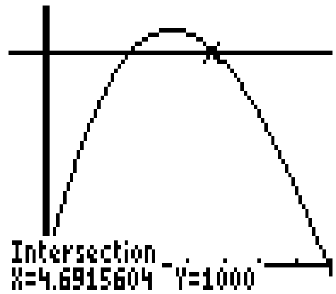
$\mathtt{y_1 = x(15.8 - 2x)(42.6 - 2x)}$ $\mathtt{y_2 = 1000}$ $[-1, 8]$ by $[-100, 1200]$



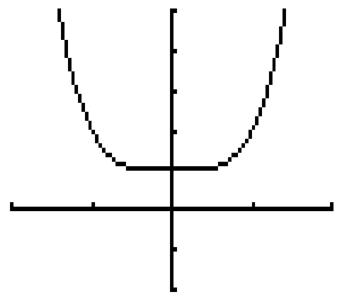


$x = 2.4115206, \quad 4.6915604$

Section 3.2

1. (a) $x^4 + 1$

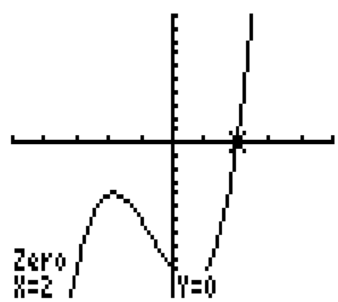
   (b) $\mathtt{y_1 = (x \wedge 6 + x \wedge 4 + x \wedge 2 + 1)/(x \wedge 2 + 1)}$
   $\mathtt{y_2 = x \wedge 4 + 1}$
   $[-2, 2]$ by $[-2, 5]$



3. (a) $P(2) = 0$, thus $(x - 2)$ is a factor of $P(x)$.

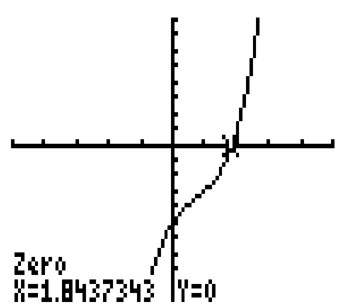
   (b) $\mathtt{y_1 = x \wedge 3 + 2x \wedge 2 - 3x - 10}$
   $[-5, 5]$ by $[-12, 10]$

   The $x-$intercept of 2 indicates $P(2) = 0$, so $x - 2$ is a factor of $P(x)$.



   (c) $\mathtt{y_1 = x \wedge 3 - 2x \wedge 2 + 3x - 5}$
   $[-5, 5]$ by $[-10, 8]$

   $x-$intercept is 1.8437, not 2.
   $x - 2$ is not a factor of $Q(x)$.



5. (a) $f(x) = x(x-1)(x-2)$

   (b) $f(x) = -x(x-1)^2(x-3)$

   (c) $f(x) = x^2(x-2)^2(x-3)$

7. (a) If $x+2$ is a factor of $f(x)$, then $f(-2) = 0$.

   Use the remainder theorem and synthetic division to find $f(-2)$.

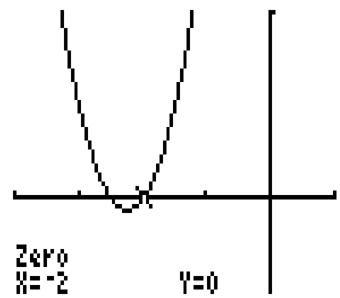
   The remainder is $9 + 2b$.

   Thus, $f(-2) = 9 + 2b = 0$, or $b = -\dfrac{9}{2}$.

   (b) $f(x) = x^2 - \left(-\dfrac{9}{2}\right)x + 5 = x^2 + \dfrac{9}{2}x + 5$

   $\mathtt{y_1 = x \wedge 2 + 9/2x + 5}$
   $[-4, 1]$ by $[-0.5, 1]$

   $x+2$ is a factor since $x = -2$ is a zero.



Section 3.3

1. The *integer* $x-$intercepts appear to be $x = -1,\ 2$.

   Use synthetic division to divide by $f(x)$ by $x+1$, getting $f(x) = (x+1)(x^3 - 6x + 4)$.

   Divide the reduced polynomial $x^3 - 6x + 4$ by $x - 2$, and we have
   $f(x) = (x+1)(x-2)(x^2 + 2x - 2)$.

   Use the quadratic formula to find the remaining zeros.

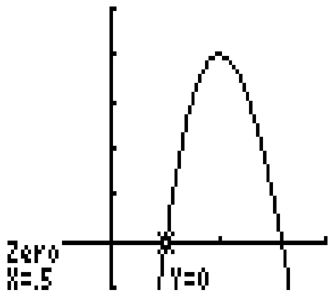
   So the zeros are $-1, 2, -1 \pm \sqrt{3}$.

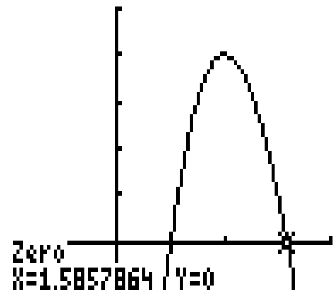
3. $V(x) = x(5-2x)(8-2x), \quad 0 < x < 2.5$

Need $x(5-2x)(8-2x) = 14$

$x-$intercept method

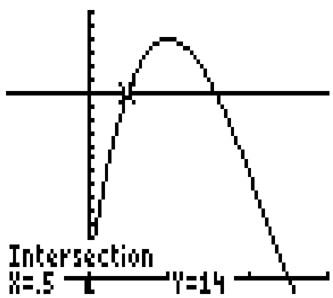
$\mathtt{y_1 = x(5-2x)(8-2x) - 14} \quad [-1, 2]$ by $[-1, 5]$

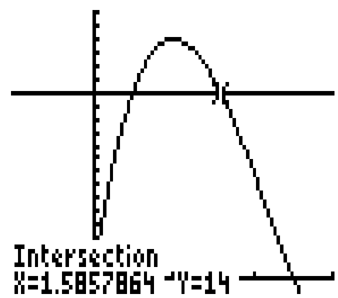




points of intersection method

$\mathtt{y_1 = x(5-2x)(8-2x),\ y_2 = 14} \quad [-1, 3]$ by $[-1, 20]$





solution

The zeros (or points of intersection) give $x = 0.5,\ 1.5857864$; thus, $x = 0.5$ or 1.59.

The corners are either 0.5 inches on a side, or 1.59 inches on a side.

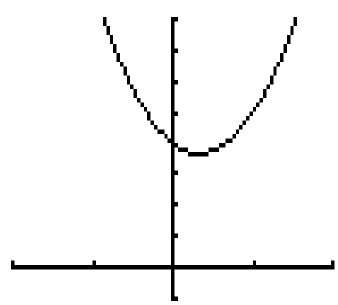
Section 3.4

1. $x = \frac{1}{3} \pm i\frac{\sqrt{11}}{3}$

$\mathtt{y_1 = 3x \wedge 2 - 2x + 4}$

$[-2, 2]$ by $[-1, 8]$

No real solutions since curve has no $x-$intercepts.



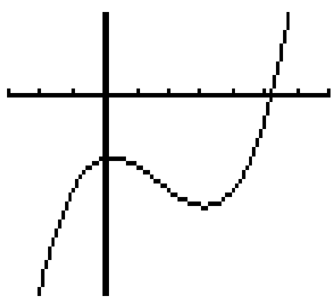
3. (a) 2 real solutions, discriminant is positive (since we have 2 $x-$intercepts)
(b) 2 nonreal complex solutions, discriminant is negative (we have no $x-$intercepts)
(c) 2 nonreal complex solutions, discriminant is negative (we have no $x-$intercepts)

Section 3.5

1. 1 real solution, 2 nonreal complex solutions

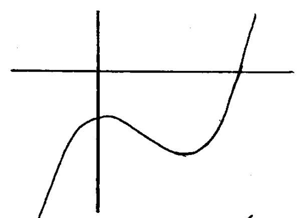
Calculator:

$\mathtt{y_1 = x \wedge 3 - 5x \wedge 2 + 2x - 16}$
$[-3, 7]$ by $[-50, 20]$



On paper:

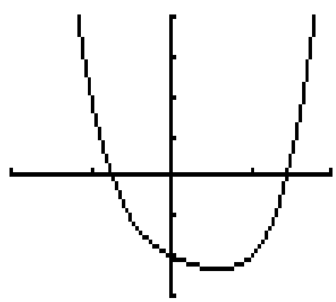
$P(x) = x^3 - 5x^2 + 2x - 16$
$y_1 = x\wedge 3 - 5x \wedge 2 + 2x - 16$
$[-3, 7]$ by $[-50, 20]$



1 real solution (1 x-intercept)
2 nonreal complex solutions
(since we know a degree 3 has 3 solutions)

3. 2 real solutions, 2 nonreal complex solution

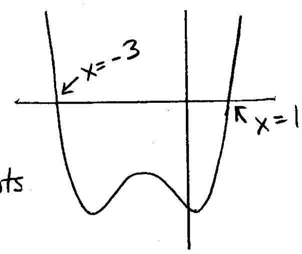
$\mathtt{y_1 = x \wedge 4 - x \wedge 3 + x \wedge 2 - x - 2}$
$[-2, 2]$ by $[-3, 4]$



5. $P(x) = (x+3)(x-1)\Big(x-(-1+i)\Big)\Big(x-(-1-i)\Big)$

On paper:

$P(x) = x^4 + 4x^3 + 3x^2 - 2x - 6$



x-intercepts appear to be $x=-3$, $x=1$ Use synthetic division to verify the intercepts and reduce the polynomial.

-3	1	4	3	-2	-6
		-3	-3	0	6
	1	1	0	-2	0

$P(x) = (x+3)(x^3 + x^2 - 2)$

1	1	1	0	-2
		1	2	2
	1	2	2	0

$P(x) = (x+3)(x-1)(x^2+2x+2)$

(x-intercepts are zeros. Zeros give factors.)

To factor x^2+2x+2, solve $x^2+2x+2=0$

$x = \dfrac{-2 \pm \sqrt{4-8}}{2} = \dfrac{-2 \pm \sqrt{-4}}{2}$

$= \dfrac{-2 \pm 2i}{2} = \boxed{-1 \pm i}$

Thus,

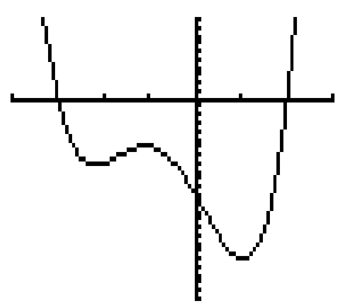
$\boxed{P(x) = (x+3)(x-1)(x-(-1+i))(x-(-1-i))}$

7. zeros: $x = -3,\ 2,\ -1 \pm i$

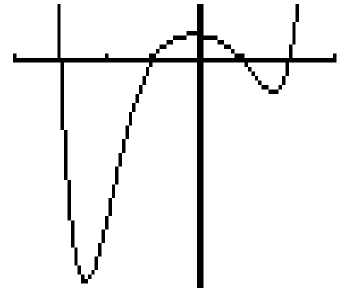
$f(x) = (x+3)(x-2)(x^2+2x+2)$

y_1 = x ∧ 4 + 3x ∧ 3 − 2x ∧ 2 − 10x − 12

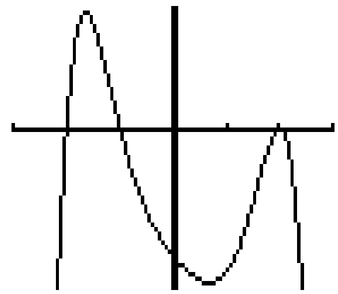
$[-4, 3]$ by $[-25, 10]$



9. zeros: $x = -3,\ -1,\ 1,\ 2,\ \pm i\sqrt{2}$
$f(x) = (x+3)(x+1)(x-1)(x-2)(x^2+2)$
$\mathtt{y_1} = \mathtt{x \wedge 6 + x \wedge 5 - 5x \wedge 4 + x \wedge 3 - 8x \wedge 2 - 2x + 12}$
$[-4, 3]$ by $[-100, 25]$



11. zeros: $x = -2,\ -1,\ 2$ (multiplicity 2), $\pm i\sqrt{2}$
$f(x) = -2(x+2)(x+1)(x-2)^2(x^2+2)$
$\mathtt{y_1} = \mathtt{-2x \wedge 6 + 2x \wedge 5 + 8x \wedge 4 - 4x \wedge 3 + 8x \wedge 2 - 16x - 32}$
$[-3, 3]$ by $[-40, 30]$



Section 3.6

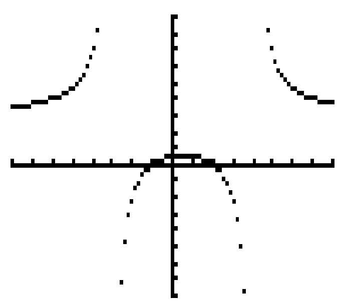
1. no $x-$intercept,
vertical asymptote is $x = 2$,
horizontal asymptote is $y = 0$

$$f(x) = \frac{1}{x-2}$$

3. $x-$intercept is 0,
vertical asymptote is $x = -2$ and $x = 2$,
horizontal asymptote is $y = 0$

$$f(x) = \frac{x}{(x-2)(x+2)}$$

5. $f(x) = \dfrac{3(x-2)(x+1)}{(x+3)(x-4)}$
$\mathtt{y_1} = \mathtt{(3(x-2)(x+1))/((x+3)(x-4))}$
$[-8, 8]$ by $[-8, 9]$
Answer may vary slightly.



7. $f(x) = \dfrac{(x+1)^2}{(x-2)}$
$\mathtt{y_1} = \mathtt{(x+1) \wedge 2/(x-2)}$
$[-6, 10]$ by $[-10, 20]$
Answer may vary slightly.