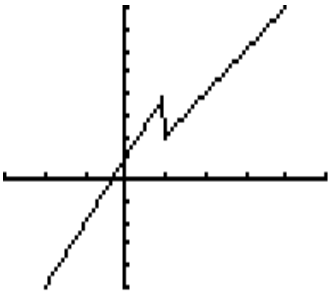
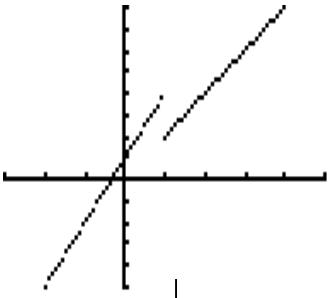
0.5 Graphing Piecewise-Defined Functions

To graph a piecewise-defined function, such as $f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$
we must specify each piece of the function and the values of x to use for that particular piece.

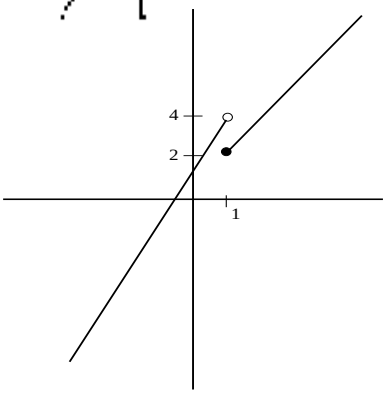
If we graph the function in the normal *connected* mode, all the points plotted will be connected by a line, such as you see here. This means the piecewise-defined function is graphed incorrectly - the left and right pieces are joined when they should not be connected unless they have a common endpoint.



Thus, we must use *dot* mode to see the correct graph. However, keep in mind that the calculator will not show what happens at the "endpoints" in the middle of the graph. We have to find the exact point for our open dot ∘ or our closed dot • by using the function definition. When $x = 1$ in the top definition $f(1) = 3(1) + 1 = 4$, so we have an open dot ∘ at $(1, 4)$ since $x = 1$ is *not* included for the top part of the definition (but that is where the graph is headed). When $x = 1$ in the second part of the definition, we have $f(1) = 2(1) = 2$, so we have a closed dot • at $(1, 2)$ since $x = 1$ *is* part of this piece of the definition. There is often a gap between the two pieces of the graph.



Thus, you must clearly indicate on paper whether each endpoint is included , using •, or not included, using ∘. You should also label the $x-$ and $y-$axes at these endpoints. So your final graph on paper should look like this:



TI-83 (see page 57), TI-89 (see page 59), TI-86 (see page 62).

Instructions for functions with three-piece definitions begin on page 64.

TI-83: Piecewise-defined Functions

To graph $f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we need to enter $y_1 = (3x+1)(x<1) + (2x)(x \geq 1)$.

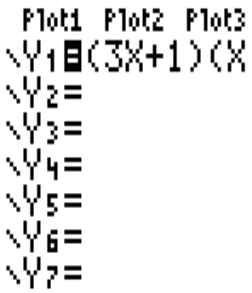
This statements means:
graph $y_1 = 3x+1$ when $x < 1$ "and" graph $y_1 = 2x$ when $x \geq 1$.

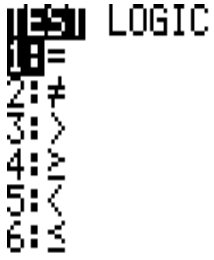
Press [Y=] and then [Clear] if a function is left over from a previous graph.

Enter `(3x+1)(x` (left picture).

To find the $<$ symbol, press [2nd] and [Test] (middle picture).
(Test is the second function for `Math`.)

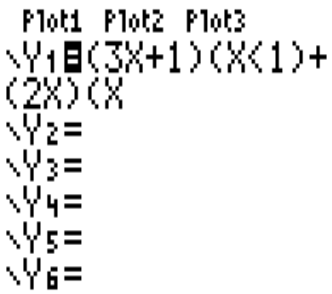
Select `5:<` by either typing 5, or using the arrow keys and pressing [Enter] (right picture).

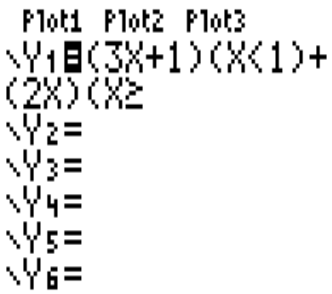




Continue typing $1) + (2x)(x$ (left picture).

Find $\geq$ by pressing [2nd] and [Test]. This time choose `4:` $\geq$ (right picture).

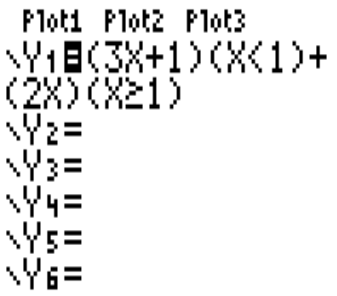


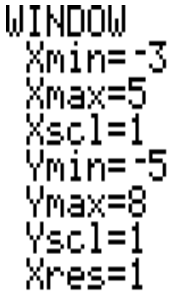


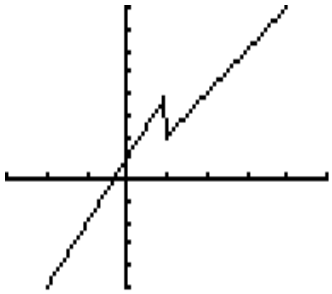
Finish typing `1)` (left picture).

Press Window and set the viewing window of $[-3, 5]$ by $[-5, 8]$ (middle picture).

Press Graph and we see this graph (right picture) since the calculator is still in *connected* mode.



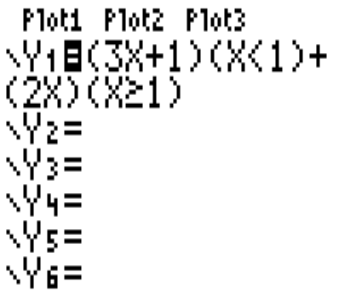


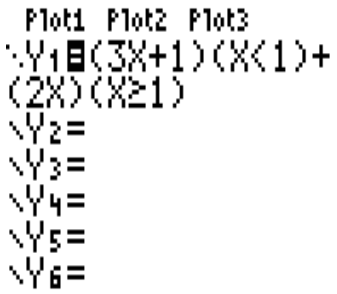


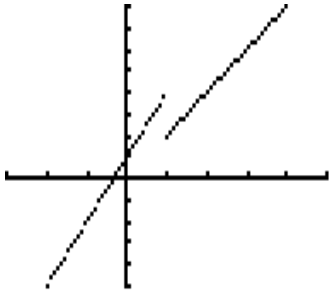
To change to *dot* mode, using the method from page 22, or from section 0.8.1 page 100, press y= to return to the function definition (left picture).

Using the left arrow key, move the cursor to the backslash to the left of y_1. Press Enter 6 times to change the backslash to a set of 3 dots (middle picture).

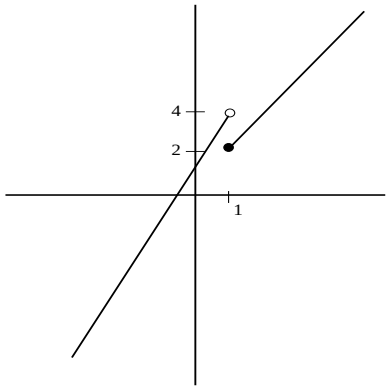
Press Graph to redraw the function using *dot* mode. Notice the gap is correct now between the endpoints (right picture).







Clearly indicating what happens at the endpoints, and labeling the $x-$ and $y-$axes for the endpoints, we have this graph when we sketch it on paper.



TI-89: Piecewise-defined Functions

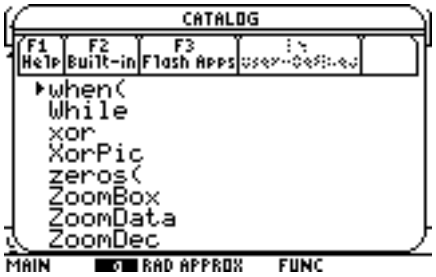
To graph the function $f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

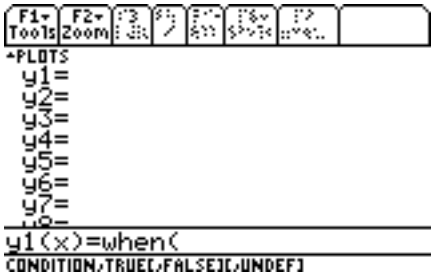
we need to tell the calculator this: $y_1 =$ when $(x < 1)$ graph $y = (3x + 1)$ and for all other values of x, graph $y = 2x$. On the calculator, it looks like y_1=when(x<1,3x+1,2x)

Press the green ♦ key and F1 Y= and Clear to erase any previous functions.

To enter the formula, we need to type when so press Catalog then type w to get the list of functions beginning with w. The first one is when((left picture).

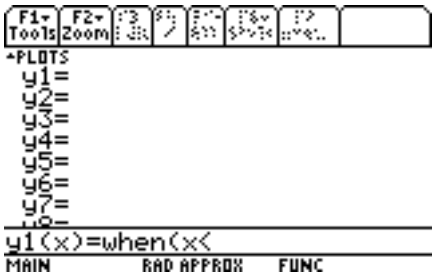
Press Enter (right picture).

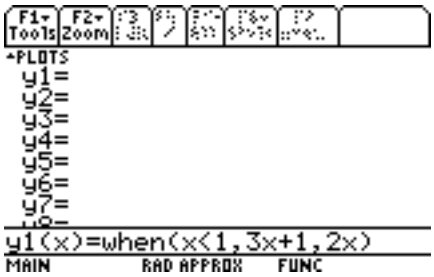




An alternate version of entering when is to press 2nd and a-lock to lock it in *alpha* mode. Type when then press alpha to turn off the alpha lock. Then type in the left parenthesis (.

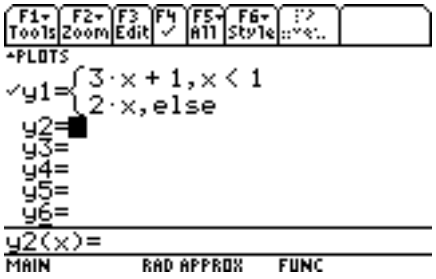
Enter x < 1, 3x + 1, 2x) to finish the definition. Type x then to find the < use 2nd and < which is the second function on 0 (left picture).
Finish the expression 1,3x+1,2x) using the comma , as needed (right picture). (Comma is above 9 .)

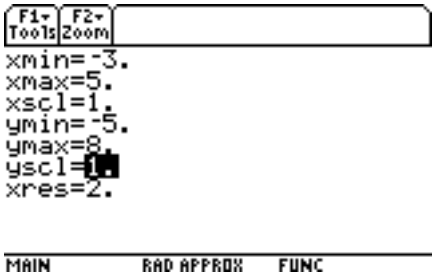




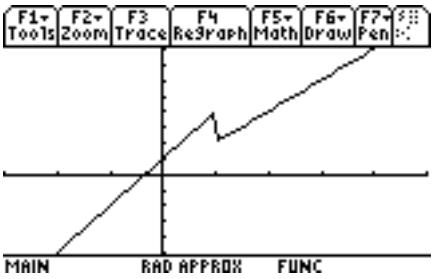
Press [Enter] and you see the piecewise definition of the function (left picture). This is actually a true-false statement which says if $x < 1$ is true, graph $y_1 = 3x + 1$, otherwise (meaning $x < 1$ is false, so when $x \geq 1$), graph $y_1 = 2x$.

Press [♦], [F2 Window] and enter the appropriate values for $[-3, 5]$ by $[-5, 8]$ (right picture).



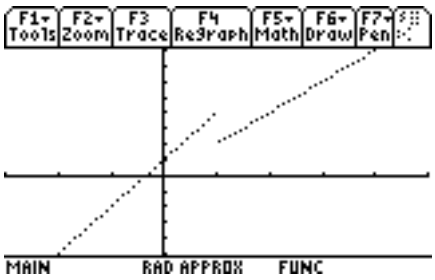


Press [♦] and [F3 Graph] to see the graph

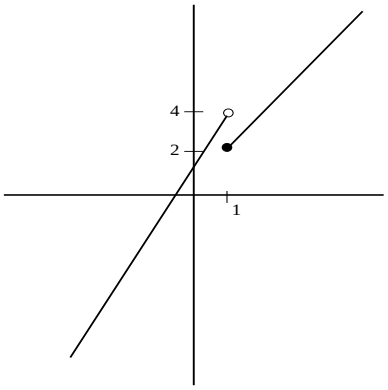


Since the calculator is in *connected* mode, the lines are joined between the left piece and the right piece, where there should be a gap. We must change to *dot* mode. Return to the definition of the function by pressing the green [♦] key and [F1 Y=]. Move the cursor to the $y_1 =$ definition and press [2nd] and [F6 Style]. We are currently in *Line* mode (see the checkmark? This is *connected* on other calculators). Select `2:Dot` by entering 2.

Press [♦] and [F3 Graph] to draw the graph again. Notice the gap between the two lines.



Clearly indicating what happens at the endpoints, and labeling the $x-$ and $y-$axes for the endpoints, we get this graph when we sketch it on paper:



For future reference, the inequality symbols can be found in several ways. Two possibilities are shown.

All four symbols $<, >, \leq, \geq$ can be found using [2nd] and [Math] (the second function on key5, then select `8:Test` to see the inequality symbols.

OR

symbol	keystrokes
$<$	[2nd], [$<$] found with [0]
$>$	[2nd], [$>$] found with [.]
$\leq$	[Catalog], [Z], scroll down until you find $\leq$
$\geq$	[Catalog], [Z], scroll down until you find $\geq$

TI-86: Piecewise-defined Functions

To graph the piecewise defined function, $f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we need to enter $y_1 = (3x+1)(x<1) + (2x)(x \geq 1)$.

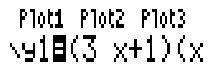
This statements means:
graph $y_1 = 3x+1$ when $x < 1$ "and" graph $y_1 = 2x$ when $x \geq 1$.

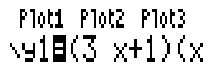
Press [Graph], then [F1] to select $y(x) =$. Press [Clear] to erase any other functions defined.

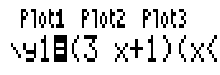
Enter `(3x+1)(x` (left picture).

To find the $<$ symbol, press [2nd] and [Test] (middle picture). (`Test` is the second function for `2`.)

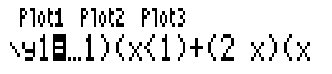
Press [F2] to select $<$ (right picture).

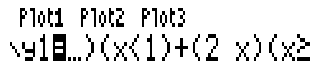


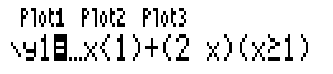




Continue typing $1) + (2x)(x$ (left picture).
Notice the inequality options for `Test` are still available (middle picture).
Press [F5] for $\geq$.
Finish typing `1)` (right picture).
Note that the line does not "wrap" as we type. Instead it scrolls to the right.



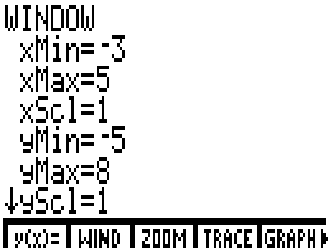


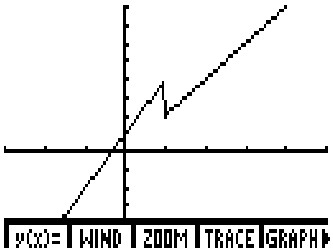


Press [Exit] to get out of the `Test` options.

Set the viewing window for $[-3, 5]$ by $[-5, 8]$ (left picture).

Press F5 Graph and we see this graph (right picture) since the calculator is still in *connected* mode.



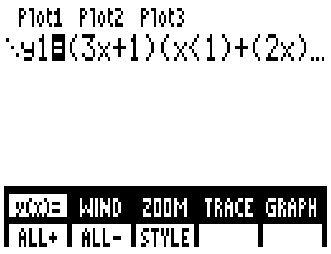


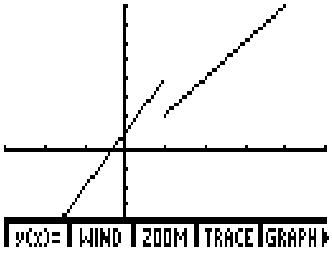
To change to *dot* mode, using the method from page 27, or from section 0.8.1 page 104, press F1 y(x)= to return to the function definition (left picture).

With the cursor on the $y_1 =$ definition, press More to see additional options. Press F3 Style 6 times, to change the backslash to the left of y_1 so it shows 3 dots (middle picture).

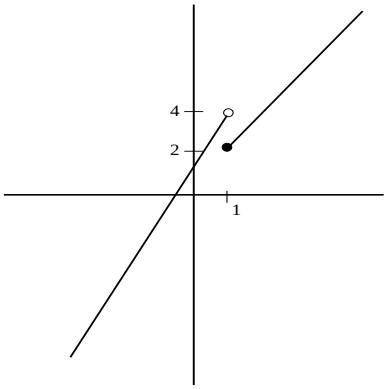
Press 2nd and F5 Graph to redraw the function using *dot* mode (right picture).

Notice the gap is correct now between the endpoints.





Clearly indicating what happens at the endpoints, and labeling the $x-$ and $y-$axes for the endpoints, we get this graph when we sketch it on paper:



A Three-part Piecewise-defined Function

Now, suppose we need to graph a function with three parts, such as

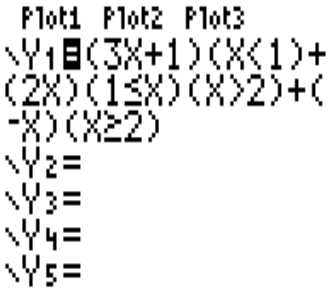
$$f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } 1 \le x < 2 \\ -x & \text{if } x \ge 2 \end{cases}$$

How do we enter this function?

TI-83 (this page), TI-89 (see page 65), TI-86 (see page 66).

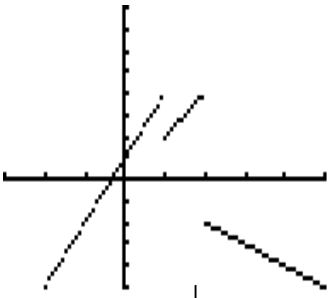
TI-83: Three-part Functions

Enter the function as
$y_1 = (3x+1)(x<1) + (2x)(1 \le x)(x<2) + (-x)(x \ge 2)$
Recall, $<$, $\le$, $>$, $\ge$ can be found using [2nd] and [Test].

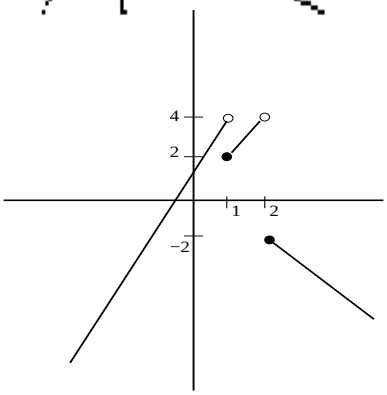


There is no shortcut for the middle portion $(2x)(1 \le x)(x < 2)$.
We cannot combine the inequalities $(1 \le x)(x < 2)$ and still get a correct graph.

Set a viewing window of $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode.



When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with $\circ$, $(1, 2)$ with $\bullet$, $(2, 4)$ with $\circ$, $(2, -2)$ with $\bullet$, and label the scale, to see



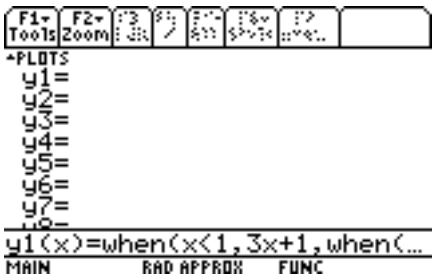
TI-89: Three-part Functions

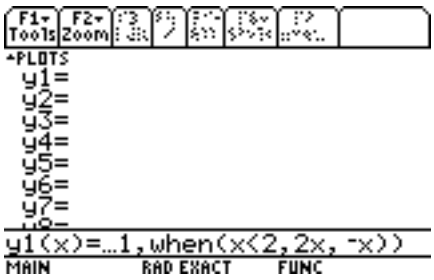
To graph $$f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } 1 \le x < 2 \\ -x & \text{if } x \ge 2 \end{cases}$$

we need to enter the function $\mathtt{y_1 = when(x < 1, 3x + 1, when(x < 2, 2x, -x))}$
(shown in two pieces below since the definition is wider than the window).

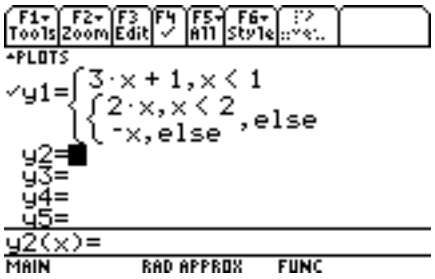
Recall $<$ can be found using [2nd] and [<], the second function on [0].

This function definition says "if $x < 1$, then use $y = 3x + 1$; otherwise if $x < 2$ (meaning $x \ge 1$ and $x < 2$), then use $y = 2x$; otherwise (meaning $x \ge 2$), use $y = -x$."



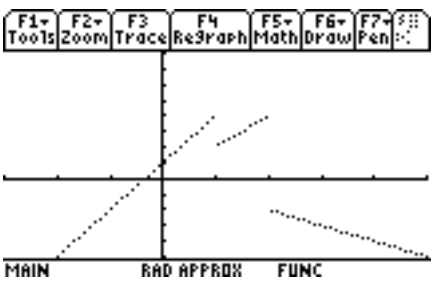


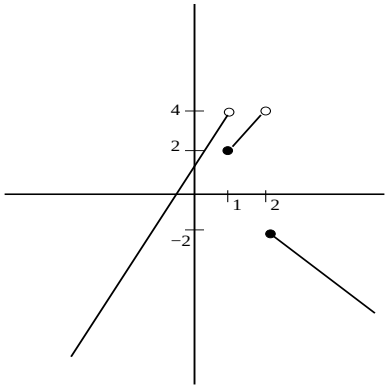
Press [Enter] and we see this function for y_1:
This looks confusing, but it says
if $x < 1$, graph $y_1 = 3x + 1$,
else we know $x \ge 1$, so within
this interval of $[1, \infty)$,
if $x < 2$, graph $y_1 = 2x$,
else graph $y_1 = -x$.



Set a viewing window for $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode (left picture).

When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with $\circ$, $(1, 2)$ with $\bullet$, $(2, 4)$ with $\circ$, $(2, -2)$ with $\bullet$, and label the scale, as shown in the right picture.



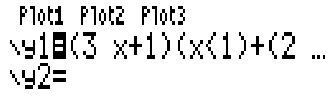


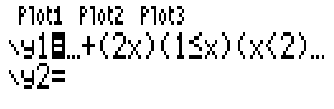
TI-86: Three-part Functions

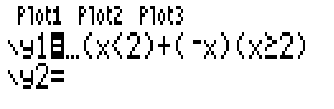
To graph $$f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2x & \text{if } 1 \le x < 2 \\ -x & \text{if } x \ge 2 \end{cases}$$

Enter the function as $\mathtt{y_1} = (\mathtt{3x+1})(\mathtt{x<1}) + (\mathtt{2x})(\mathtt{1 \le x})(\mathtt{x<2}) + (\mathtt{-x})(\mathtt{x \ge 2})$

To find the $<, \le, >, \ge$ symbols, press [2nd] and [Test] (the second function for 2).
(The function is shown here in three pieces since the definition is wider than one line.)



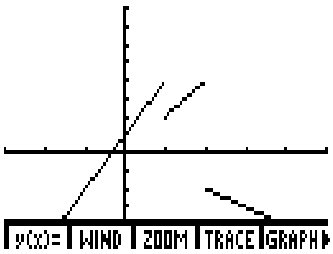




There is no shortcut for the middle portion $(2x)(1 \le x)(x < 2)$.
We cannot combine the inequalities $(1 \le x)(x < 2)$ and still get a correct graph.

Press [Exit] to get rid of the `Test` menu.

Set a viewing window of $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode.



When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with $\circ$, $(1, 2)$ with $\bullet$, $(2, 4)$ with $\circ$, $(2, -2)$ with $\bullet$, and label the scale, to see