0.3 Graphing Algebraic Functions

There are some special functions we need to be able to graph easily.

Directions follow for

- rational functions (see page 21),
- absolute value functions (see page 28),
- circles (see page 33),
- radical functions (see page 38),

0.3.1 Rational Functions

To graph rational functions, we must carefully use parentheses for the numerator and denominator when they involve more than one character. Read/review section 0.1.3 (How to Enter Expressions).

Let's graph $y = \dfrac{2x}{x+3}$.

We know the graph has a vertical asymptote of $x = -3$, since -3 makes the denominator equal to 0, and thus, the function is undefined. Therefore, the vertical line $x = -3$ is not part of the graph and should be drawn as a dashed line when we copy our graph to paper.

When graphing this on the calculator, we will

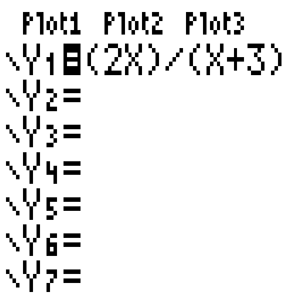
- Enter the function formula, `y=(2x)/(x+3)`.
  Notice the parentheses around the numerator and denominator.
- Choose the viewing window, such as $[-10, 5]$ by $[-8, 8]$, and adjust the window if necessary.
- Select *dot* mode instead of connected mode for the curve drawn.
- Draw the graph.
- When copying the graph to paper, include the vertical asymptote as a dashed line. After studying rational functions more, we will also include the horizontal asymptote as a dashed line.

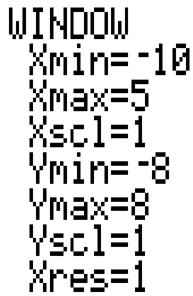
TI-83 (see page 22), TI-89 (see page 24), TI-86 (see page 26).

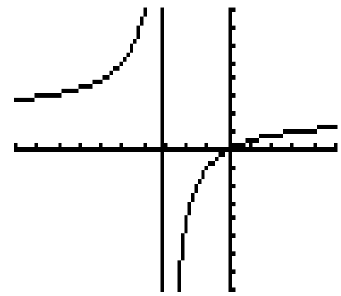
TI-83 Graphing Rational Functions

Press [Y=] and [Clear] to erase any previous functions.

Enter `(2x)/(x+3)` for y_1 (left picture). Enter the viewing window $[-10, 5]$ by $[-8, 8]$ (middle picture). Draw the graph by pressing [Graph] (right picture).

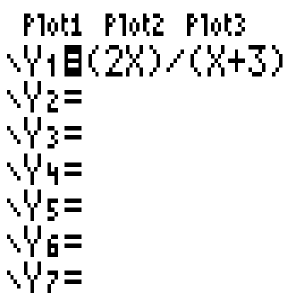


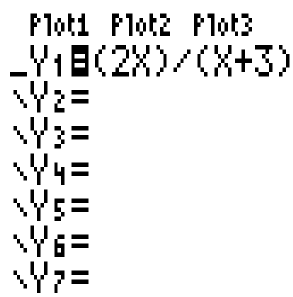




The graph is currently drawn using *connected* mode. The calculator connects the points it plots, so the top point of the curve on the left is connected to the bottom point of the curve on the right, creating the vertical line, which makes the line $x = -3$ look like it is part of the graph proper. We do not want this line as part of our graph.

To change to *dot* mode, press [Y=] to return to the function definition. Notice the backslash in front of y_1 (left picture) ? Using the left arrow key, move the cursor to the left of y_1, so the cursor is blinking on top of the backslash (right picture).

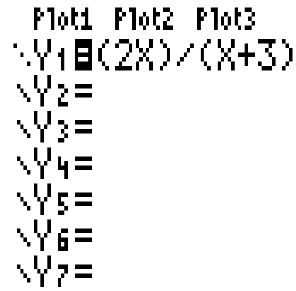




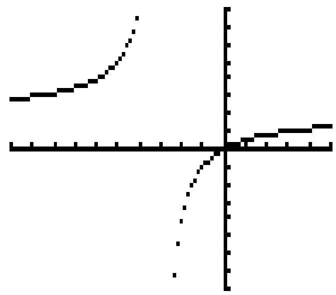
Press [Enter] 6 times to make the backslash a set of 3 dots. The backslash will change to

1. a thick line - used for drawing very thick curves
2. an upper triangle - used for shading above curves
3. a lower triangle - used for shading below curves
4. a line with an open circle
5. an open circle
6. a dotted line - used for drawing a dotted line instead of a solid line
   (This is the one we want.)

Move the cursor to the right of y_1 with the right arrow key.

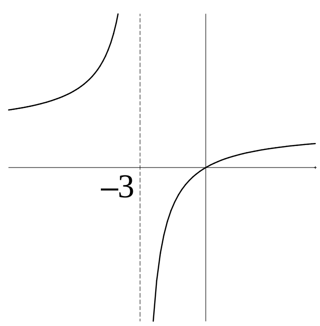


Press Graph to see



This is a more appropriate graph for $y = \dfrac{2x}{x+3}$. When copying this graph on paper, draw the curves with solid lines, and draw in the vertical line $x = -3$ as a dashed line. Also label the scale on the $x-$axis for the vertical asymptote. (After working with the section on graphing rational functions, you will also be able to find the horizontal asymptote and include it in your graph as well.)

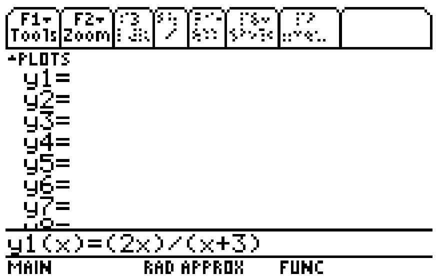
For now, we might draw this:

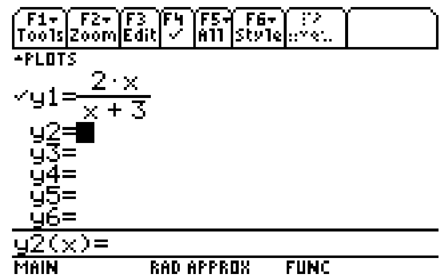


TI-89 Graphing Rational Functions

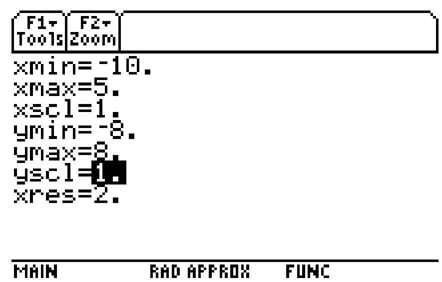
We want to graph $y = \dfrac{2x}{x+3}$.

Press ♦ and F1 Y= . Enter `(2x)/(x+3)` for y_1 (left picture). Pressing Enter , we see the function correctly defined (right picture).

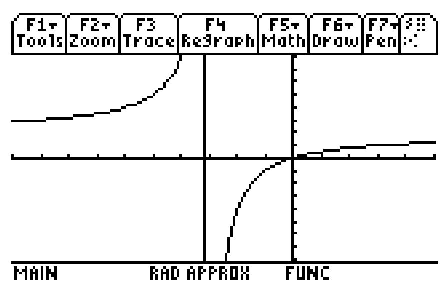




Using ♦ and F2 Window ,
enter the viewing window $[-10, 5]$ by $[-8, 8]$



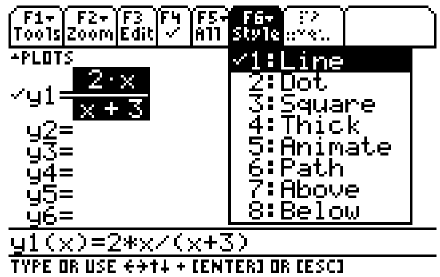
Draw the graph by pressing ♦
and F3 Graph :



The graph is currently drawn using *connected* mode. The calculator connects the points it plots, so the top point of the curve on the left is connected to the bottom point of the curve on the right, creating the vertical line, which makes the line $x = -3$ look like it is part of the graph proper. We do not want this line as part of our graph.

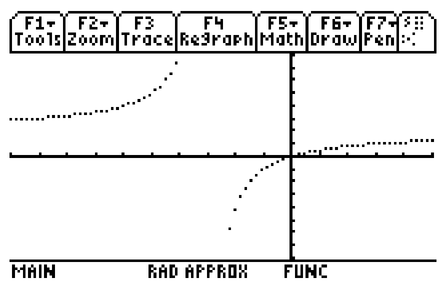
To change to *dot* mode, press ♦ and F1 Y= to return to the function definition. (Also see section 0.8.1.)

With the cursor on the function definition, press 2nd and F6 Style .



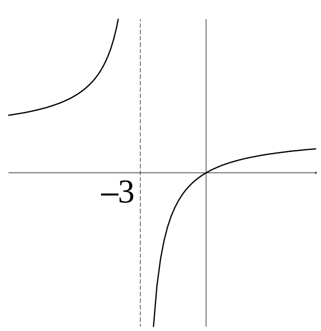
See the checkmark beside `1:Line`? This means we are using *connected*, or *Line*, mode for drawing the graph. Enter the number 2, or using the down arrow key, move the cursor to `2:Dot` and press Enter .

Now redraw the graph using ♦ and F3 Graph



This is a more appropriate graph for $y = \dfrac{2x}{x+3}$. When copying this graph on paper, draw the curves with solid lines, and draw in the vertical line $x = -3$ as a dashed line. Also label the scale on the $x-$axis for the vertical asympotote. (After working with the section on graphing rational functions, you will be able to find the horizontal asymptote and include it in your graph.)

For now, we might draw this:

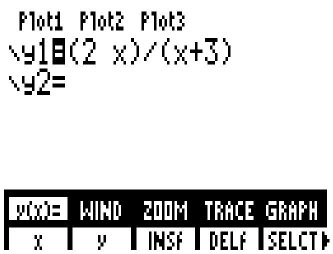


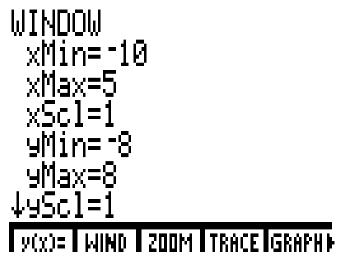
TI-86 Graphing Rational Functions

We want to graph $y = \dfrac{2x}{x+3}$.

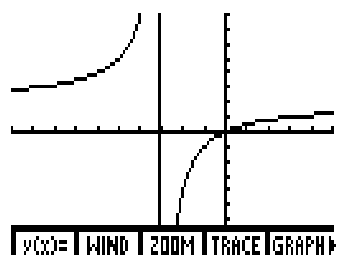
Press Graph and F1 y(x)= . Clear any previously defined functions.

Enter `(2x)/(x+3)` for y_1 (left picture) and the viewing window of $[-10, 5]$ by $[-8, 8]$ (right picture).





Draw the graph by pressing F5 Graph .



The graph is currently drawn using *connected* mode. The calculator connects the points it plots, so the top point of the curve on the left is connected to the bottom point of the curve on the right, creating the vertical line, which makes the line $x = -3$ look like it is part of the graph proper. We do not want this line as part of our graph.

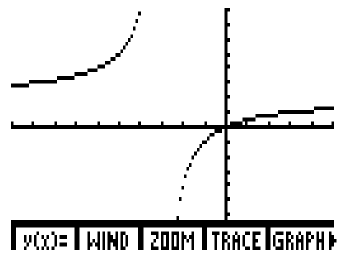
To change to *dot* mode, press F1 y(x)= to return to the function definition. With the cursor on the definition y_1 for this function (left picture), press More to see the additional options available (right picture). Notice the backslash in front of y_1? It is currently indicating we are using a line to draw the graph.

Press F3 Style and notice the change in the backslash.

Each time you press F3 Style the backslash changes to the following:

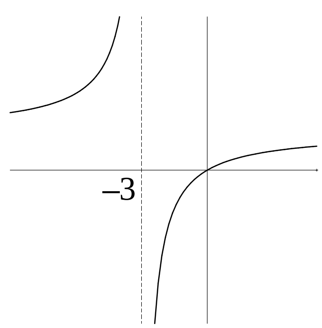
1. a thick line - used for drawing very thick curves
2. an upper triangle - used for shading above curves
3. a lower triangle - used for shading below curves
4. a line with an open circle
5. an open circle
6. a dotted line - used for drawing a dotted line instead of a solid line

This last one, the backslash as three dots, is the one we want for *dot* mode (left picture). Press 2nd and F5 Graph (right picture).



This is a more appropriate graph for $y = \dfrac{2x}{x+3}$. When copying this graph on paper, draw the curves with solid lines, and draw in the vertical line $x = -3$ as a dashed line. Also label the scale on the $x-$axis for the vertical asymptote. (After working with the section on graphing rational functions, you will also be able to find the horizontal asymptote and include it in your graph as well.)

For now, we might draw this:



0.3.2 Absolute Value Functions

Let's graph $y = |x - 1| + 2$.

We must:

- Find the *absolute value* function in the calculator menu of functions.
- Enter the expression for the y, using the absolute value function `abs( )`.
- Select a viewing window and graph the function.
- Adjust the viewing window as needed.

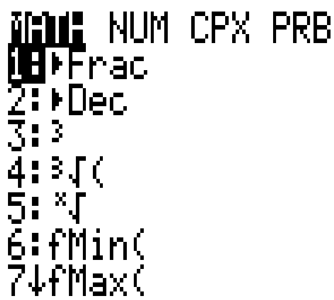
TI-83 (see page 29), TI-89 (see page 30), TI-86 (see page 32).

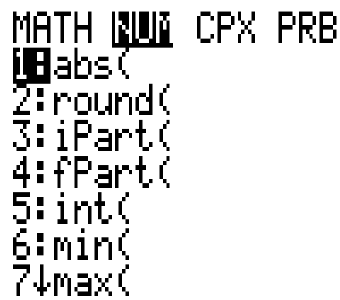
TI-83: Absolute Value Functions

To graph $y = |x - 1| + 2$, press (Y=) and then (Clear) if a function is left over from a previous graph.

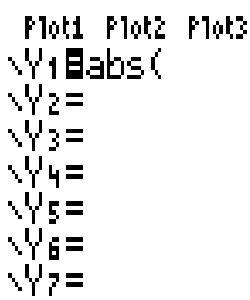
To enter the formula for y_1, we need the absolute value function.

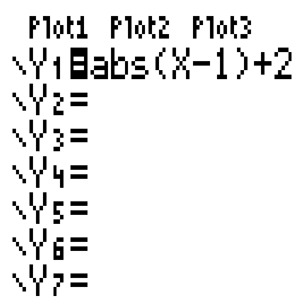
Press (Math) to see the left picture. Use the right arrow to move to `NUM` as shown in the right picture.



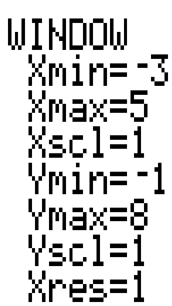


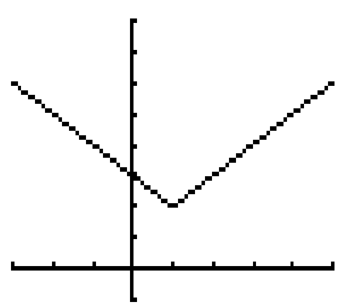
Press (Enter) to select the absolute value function `abs` (left picture).
Type in `x-1)+2` to finish the definition so we have the right picture.





Enter a viewing window of $[-3, 5]$ by $[-1, 8]$ (left picture).
Press (Graph) (right picture).



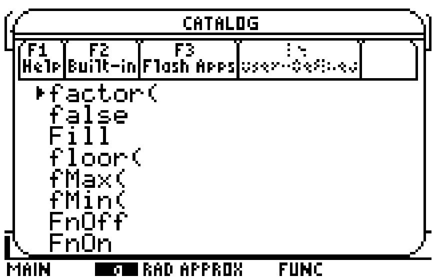


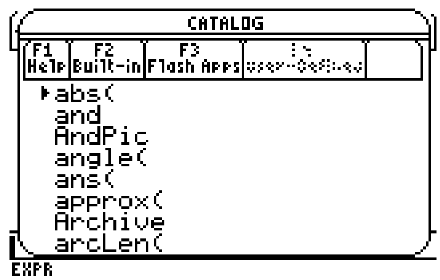
TI-89: Absolute Value Functions

To graph $y = |x-1|+2$, press the green ♦ key and then F1 Y= . If a function is left over from a previous graph, put the cursor on the function definition, and press Clear .

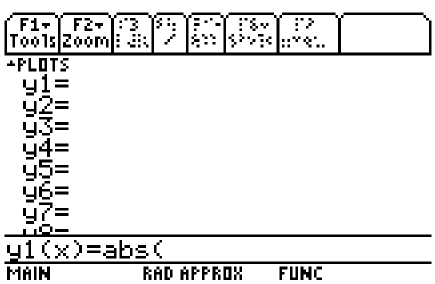
To enter the formula $|x-1|+2$ for y_1, we need the absolute value function.

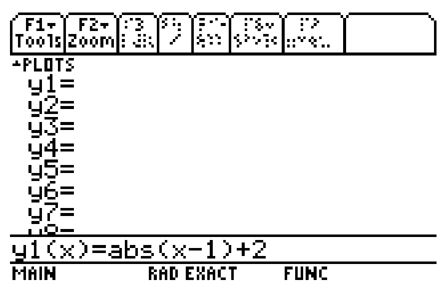
Press Catalog to see an alphabetical list of functions. You may see a different set of functions, based on what function you last searched for using this feature (left picture). Hold down alpha and press A to see the functions beginning with the letter a (right picture).



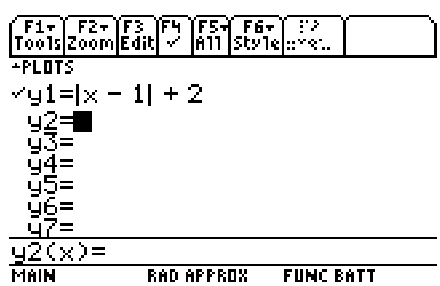


Since absolute value, `abs`, is the first choice, press Enter to select it (left picture).
Type `x-1)+2` to finish the function definition (right picture).

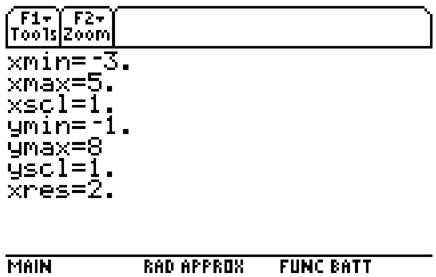


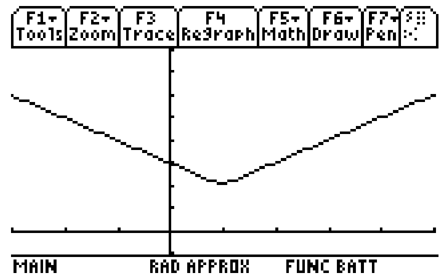


Press Enter and check to be sure the function is correct.



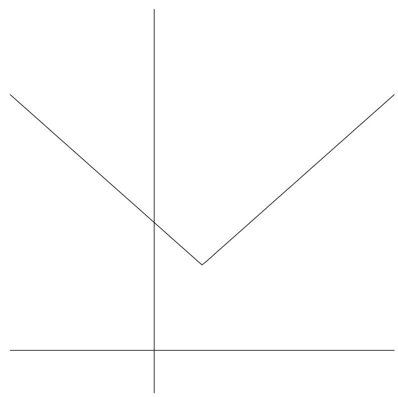
Enter a viewing window of $[-3, 5]$ by $[-1, 8]$ (left picture). Press ♦ and F3 Graph (right picture).





When transferring this graph to paper, we know the absolute value gives a "V-shaped" graph, so the function is not curved at the minimum value, but has a sharp "point".

Thus, the function looks more like

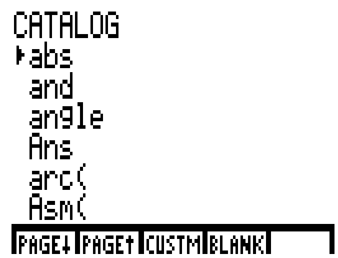


TI-86: Absolute Value Functions

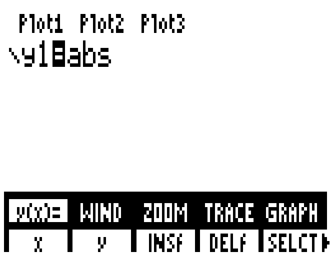
To graph $y = |x - 1| + 2$, press [Graph], then [F1 y(x)=]. Clear any previously defined functions.

To enter the formula $|x - 1| + 2$ for y_1, we need the absolute value function.

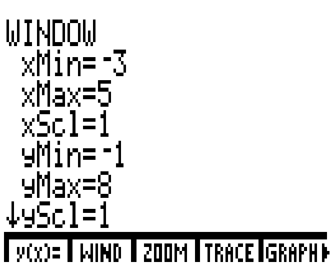
Press [2nd] and [Catlg-Vars] (left picture). Press [F1 Catlg] (middle picture). If you do not see the functions starting with the letter a, press [Alpha], followed by [A], or you can use [F1 Page down] or [F2 Page up] until you find `abs` (right picture).

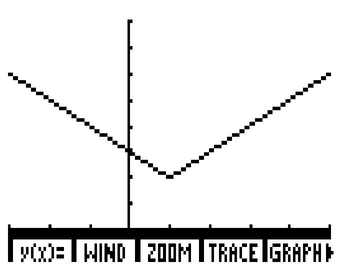


Press [Enter] to select the absolute value function `abs` (left picture).
Type in `(x-1)+2` to finish the definition (right picture).



Enter a viewing window of $[-3, 5]$ by $[-1, 8]$ (left picture). Press [F5 Graph] (right picture).





0.3.3 Circles

We need to understand how to graph a circle, such as $x^2 + y^2 = 6$. First, find the center and the radius of the circle. The center is $(0, 0)$ and the radius is $\sqrt{6}$ which is less than 3.

Since the calculator can only graph functions $y = ...,$ we must first solve the equation for y. Do this algebraically before you continue reading.

Given $x^2 + y^2 = 6$ solve for y.

Did you get $y = \pm\sqrt{6 - x^2}$? We need to graph $y = \sqrt{6 - x^2}$ and $y = -\sqrt{6 - x^2}$ on the same graph. We can use a window of $[-3, 3]$ by $[-3, 3]$ to see the circle, since the radius is less than 3.

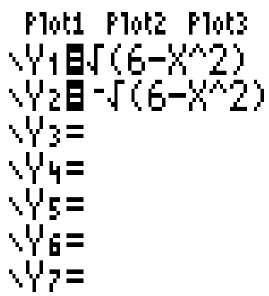
Continue reading for *your* calculator to learn how to make the graph look round like a circle instead of an oval. The graph should also be a connected curve, not two separate pieces when you draw it on paper. The $x-$intercepts are the values on the $x-$axis where the top half of the circle and the bottom half of the circle will connect.

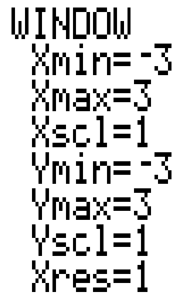
TI-83 (see page 34), TI-89 (see page 35), TI-86 (see page 37).

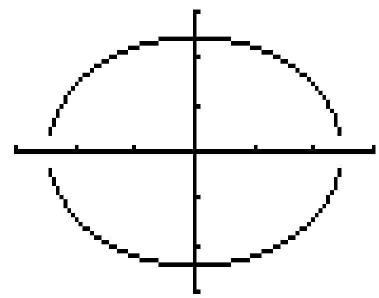
TI-83: Circles

To graph $x^2 + y^2 = 6$ by using $y_1 = \sqrt{6 - x^2}$ and $y_2 = -\sqrt{6 - x^2}$, press Y= and Clear if a function is left over from a previous graph.

Enter the two function formulas for y_1 and y_2 (left picture). Be careful to use the negative sign, not subtraction, in front of the square root. Press Window and enter values for $[-3, 3]$ by $[-3, 3]$ (middle picture). Press Graph (right picture).

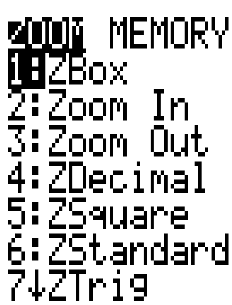


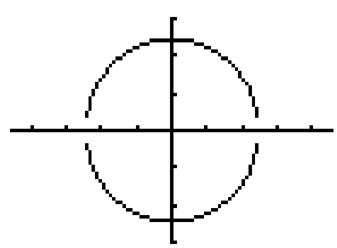




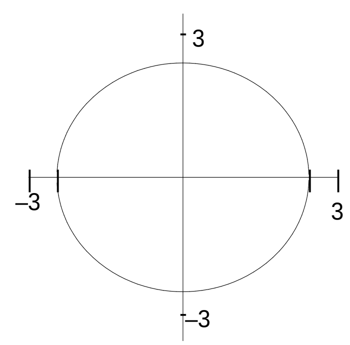
The graph of $y = \sqrt{6 - x^2}$ is the upper semi-circle since the $y-$values are positive and $y = -\sqrt{6 - x^2}$ is the lower semi-circle since the $y-$values are negative. This looks more like an ellipse than a circle since the one unit intervals on the $x-$axis and the $y-$axis are not the same. To get a graph that looks more appropriate, we can use the `zoom square` feature.

Press Zoom (left picture). Select `5:ZSquare` for zoom square (right picture).





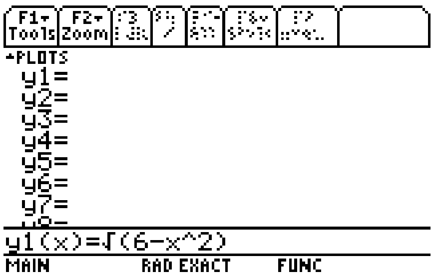
The graph looks more like a circle now, but the curves do not meet at the $x-$axis. The gap at the $x-$axis is the best the calculator can do. We know the two semi-circles should be connected, since a circle is one continuous curve. We must connect the the two semi-circles when we draw the graph on paper.

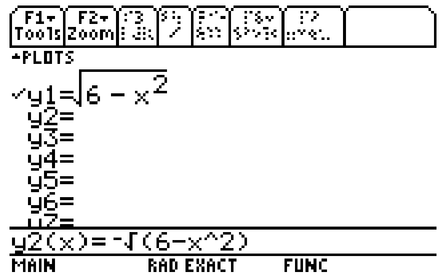


TI-89: Circles

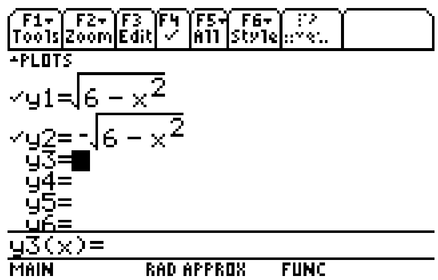
To graph $x^2 + y^2 = 6$ by using $y_1 = \sqrt{6 - x^2}$ and $y_2 = -\sqrt{6 - x^2}$, press the green ◆ key and F1 Y= . If a function is left over from a previous graph, put the cursor on the function definition, and press Clear .

Enter these two functions for y_1 and y_2, being careful to use the negative sign, not subtraction, in front of the square root.

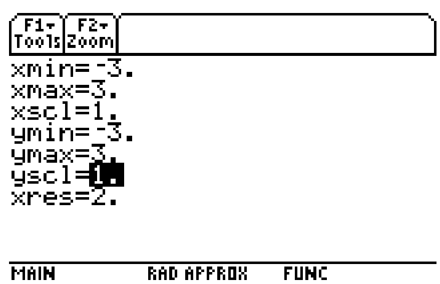




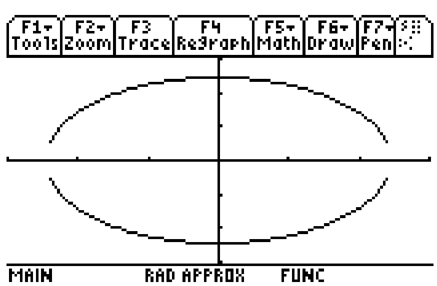
Press Enter and verify the functions are correct.



Press ◆ and F2 Window . Enter values for $[-3, 3]$ by $[-3, 3]$



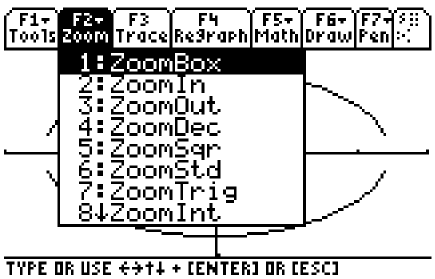
Press ◆ and F3 Graph .

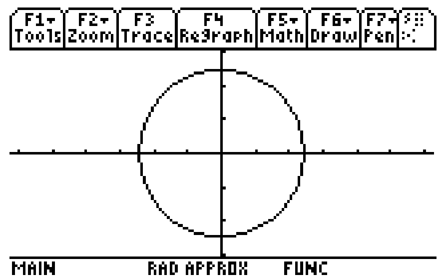


The graph of $y = \sqrt{6 - x^2}$ is the upper semi-circle since the $y-$values are positive and $y = -\sqrt{6 - x^2}$ is the lower semi-circle since the $y-$values are negative. This looks more like an ellipse than a circle since the one unit intervals on the $x-$axis and the $y-$axis are not the same.

To get a graph that looks more appropriate, we can use the `zoom square` feature.

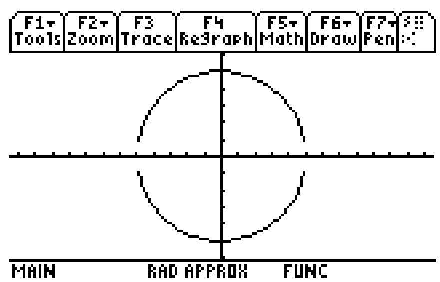
Press F2 Zoom (left picture). Select `5:ZoomSqr` (right picture).



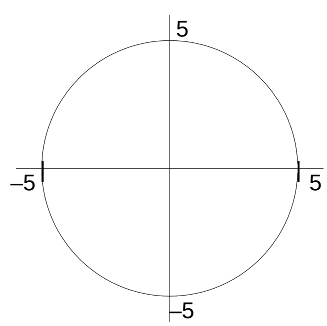


The graph looks more like a circle now. On this graph, the curves actually appear to be joined at the $x-$axis, which is correct.

Often the semi-circles do not appear to be joined at the $x-$axis. For example, if we graphed the circle $x^2 + y^2 = 25$ with `zoom square`, we get



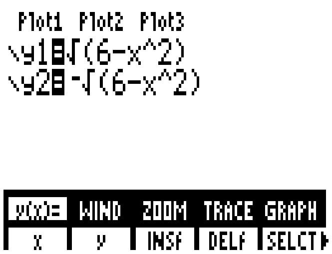
Leaving the gap is the best the calculator can do. We know the two semi-circles should be connected, since a circle is one continous curve. We must connect the two semi-circles when we draw the graph on paper. So, this circle, $x^2 + y^2 = 25$, should look like

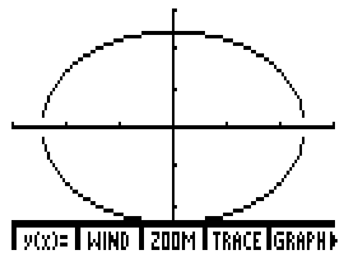


TI-86: Circles

To graph $x^2 + y^2 = 6$ by using $y_1 = \sqrt{6 - x^2}$ and $y_2 = -\sqrt{6 - x^2}$, press Graph and F1 y(x)= . Press Clear for any functions left over from a previous graph.

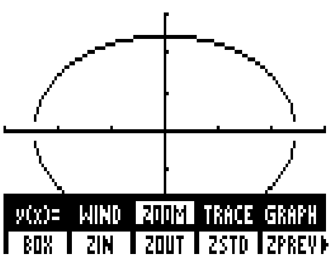
Enter the two function formulas for y_1 and y_2, using parentheses appropriately (left picture). Be careful to use the negative sign, not subtraction, in front of the square root. Press 2nd and F2 Wind and enter values for $[-3, 3]$ by $[-3, 3]$ (middle picture). Press F5 Graph (right picture).

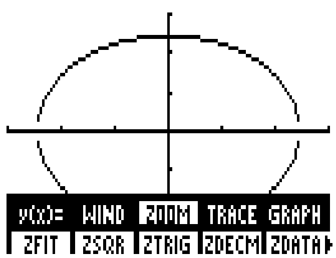


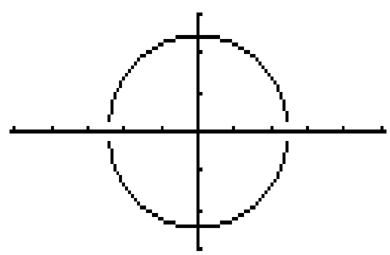


The graph of $y = \sqrt{6 - x^2}$ is the upper semi-circle since the $y-$values are positive and $y = -\sqrt{6 - x^2}$ is the lower semi-circle since the $y-$values are negative. This looks more like an ellipse than a circle since the one unit intervals on the $x-$axis and the $y-$axis are not the same. To get a graph that looks more appropriate, we can use the `zoom square` feature.

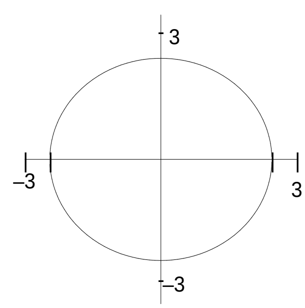
Press F3 Zoom (left picture), and More to see additional zoom features (middle picture). Press F2 ZSqr for zoom square, and Clear to get rid of the menu at the bottom (right picture). (Press Graph to display the menu again.)







The graph looks more like a circle now, but the curves do not meet at the $x-$axis. This is the best the calculator can do. Since a circle is one continuous curve, we must connect the two semi-circles when we draw the graph on paper.



0.3.4 Radical Functions

Read/review section 0.1 (Entering Expressions Correctly).

When graphing functions containing radical expressions, we must carefully enter the expression, including the appropriate parentheses.

Since the square root is the only radical with a special key on the calculator, we will graph examples of other roots here as well as a square root.

We will graph 1) $y = x^{2/3}$,
2) $y = \sqrt[4]{x-1} - x + 3$,
and 3) $y = \sqrt{5-3x}$.

In each case, first find the domain for the function, to get an idea of an appropriate viewing window.

Given $y = x^{2/3}$, we know this is the same as $y = \sqrt[3]{x^2}$, so the domain is $(-\infty, \infty)$. (We can take odd roots of any number.) The $y-$values are always positive due to the x^2, so let's try a window of $[-3, 3]$ by $[-1, 5]$. The $x-$intercept is $(0, 0)$, so we know the graph does touch the origin.

Given $y = \sqrt[4]{x-1} - x + 3$, we know $x - 1 \geq 0$ (since we have an even root), so the domain is $[1, \infty)$. Let's try a window of $[-1, 8]$ by $[-5, 5]$. (I'm including a *little* "whitespace" to the left of the function to get a better view.)

Given $y = \sqrt{5-3x}$, we know $5 - 3x \geq 0$ (since we have an even root), which yields $x \leq \dfrac{5}{3}$, so the domain is $\left(-\infty, \dfrac{5}{3}\right]$. Let's try a window of $[-5, 2]$ by $[-1, 6]$.
Find the $x-$intercept to determine if the graph touches the $x-$axis (let $y = 0$ and solve for x, or in this case, we know $y = 0$ when $5 - 3x = 0$ since that gives us $\sqrt{0}$. So, solve $5 - 3x = 0$ to find the $x-$intercept.)

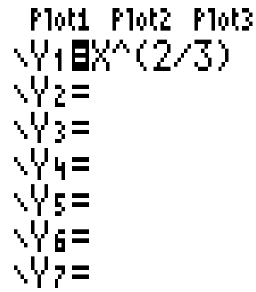
TI-83 (see page 39), TI-89 (see page 41), TI-86 (see page 44).

TI-83: Radical Functions

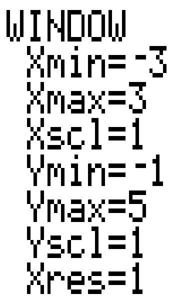
Graph $y = x^{2/3}$ with domain $(-\infty, \infty)$, and a window of $[-3, 3]$ by $[-1, 5]$.

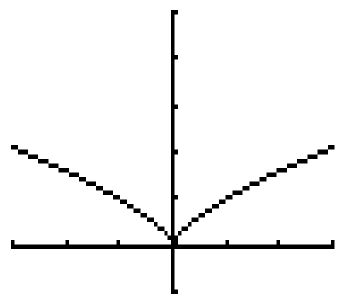
To start the graph, press [Y=] and then [Clear] if a function is left over from a previous graph.

Enter the function formula, using parentheses around the exponent.



Press [Window] and enter values for $[-3, 3]$ by $[-1, 5]$ (left picture). Press [Graph] (right picture).



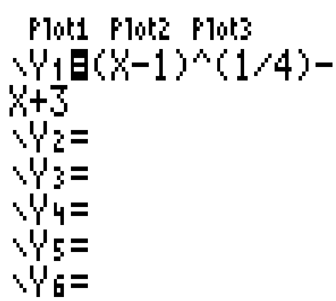


Notice, the point $(0, 0)$ is part of the graph, since it is the $x-$intercept.

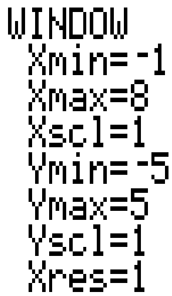
Next, graph $y = \sqrt[4]{x-1} - x + 3$ with a domain of $[1, \infty)$, and a window of $[-1, 8]$ by $[-5, 5]$.

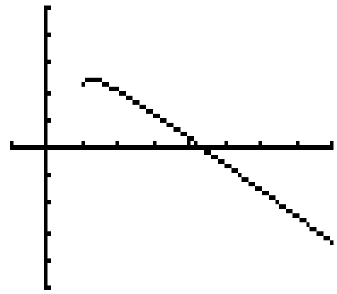
Press [Y=] and [Clear] the previous function.

Enter the function formula, using the exponent of $\frac{1}{4}$ and parentheses appropriately.



Set the window for $[-1, 8]$ by $[-5, 5]$ (left picture). Press Graph (right picture).

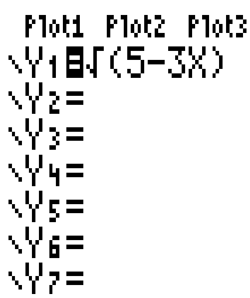


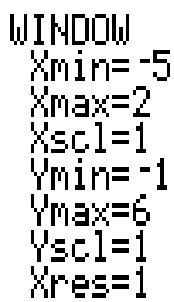


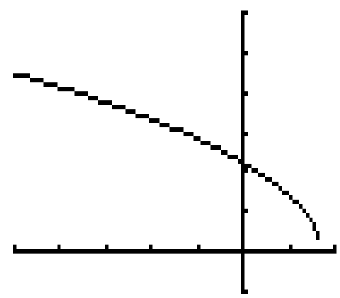
The domain is $[1, \infty)$. We also know when $x = 1$, $y = 2$, so the graph begins at the point $(1, 2)$.

Finally, we graph $y = \sqrt{5 - 3x}$ with a domain of $\left(-\infty, \frac{5}{3}\right]$, and a window of $[-5, 2]$ by $[-1, 6]$. Press Y= and Clear the previous function.

Enter the function formula and appropriate parentheses (left picture). Set the window for $[-5, 2]$ by $[-1, 6]$ (middle picture). Press Graph (right picture).

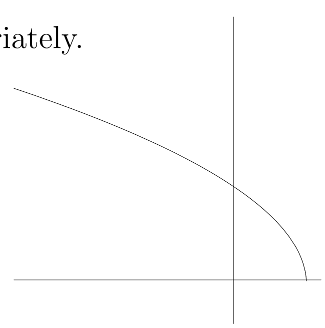






Now a question arises. Does the graph touch the $x-$axis? We have to determine the answer ourself, since the calculator graph cannot make it clear.

Find the $x-$intercept: $y = 0$ when $\sqrt{5 - 3x} = 0$ which happens when $5 - 3x = 0$, or $x = \frac{5}{3}$.
Thus, the point $\left(\frac{5}{3}, 0\right)$ *is* part of our graph. When we draw the graph on paper, we must connect the curve to the $x-$axis appropriately.

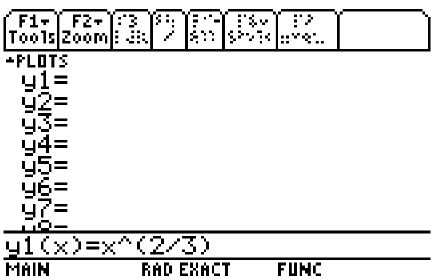


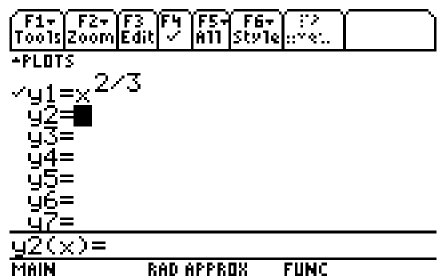
TI-89: Radical Functions

Graph $y = x^{2/3}$ with domain $(-\infty, \infty)$ and a window of $[-3, 3]$ by $[-1, 5]$.

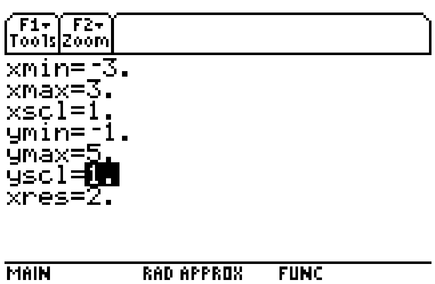
Press the green ♦ key and F1 Y= . If a function is left over from a previous graph, put the cursor on the function definition, and press Clear .

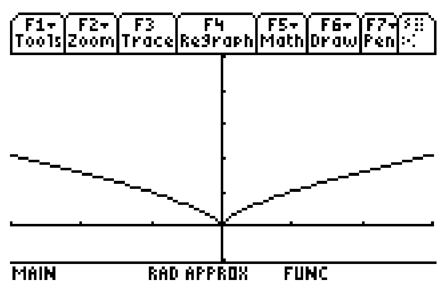
Enter the new definition (left picture). Press Enter and verify the formula is correct (right picture).





Set the window for $[-3, 3]$ by $[-1, 5]$, using ♦ and F2 Window (left picture). Press ♦ and F3 Graph (right picture).



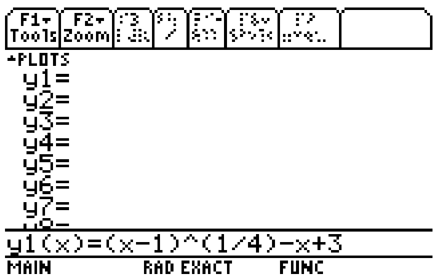


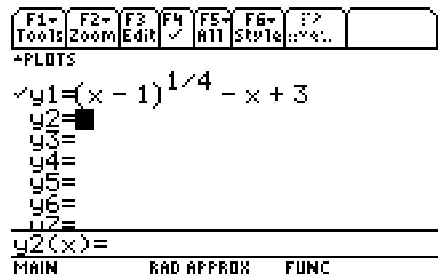
Notice, the point $(0, 0)$ is part of the graph, since it is the $x-$intercept.

Graph $y = \sqrt[4]{x-1} - x + 3$ with a domain of $[1, \infty)$ and a window of $[-1, 8]$ by $[-5, 5]$.

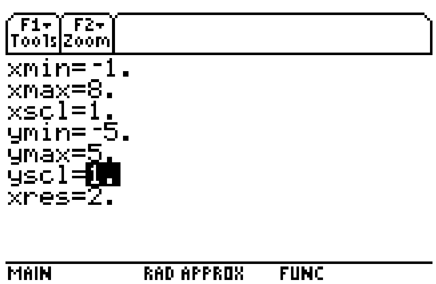
Press ♦ key and F1 Y=. Move the cursor up to y_1 and press Clear.

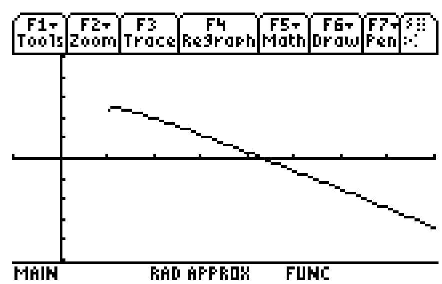
Enter the new definition, using the exponent of $\frac{1}{4}$ and parentheses appropriately (left picture). Press Enter and verify the formula is correct (right picture).





Set the window for $[-1, 8]$ by $[-5, 5]$, using ♦ and F2 Window (left picture). Press ♦ and F3 Graph (right picture).



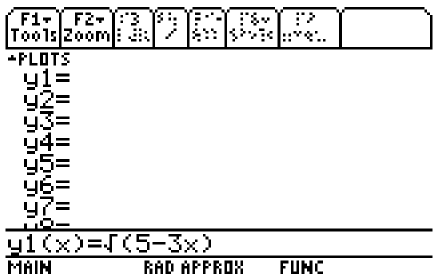


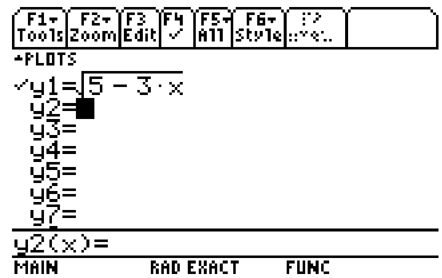
The domain is $[1, \infty)$. We also know when $x = 1$, $y = 2$, so the graph begins at the point $(1, 2)$.

Finally, we graph $y = \sqrt{5 - 3x}$ with a domain of $\left(-\infty, \frac{5}{3}\right]$, and a window of $[-5, 2]$ by $[-1, 6]$.

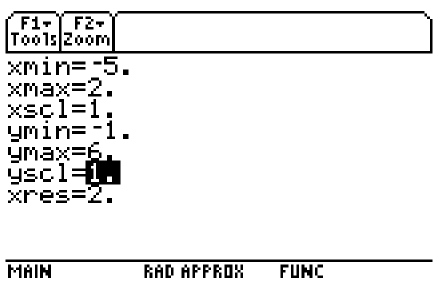
Press ♦ key and F1 Y=. Move the cursor up to y_1 and press Clear to erase the previous function.

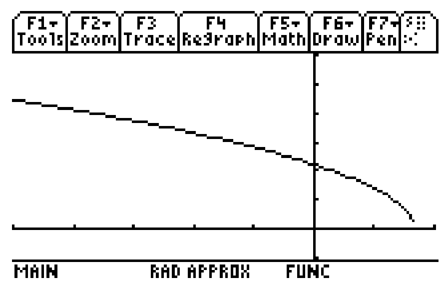
Enter the new definition (left picture). Press [Enter] and verify the formula is correct (right picture).





Set the window for $[-5, 2]$ by $[-1, 6]$ (left picture). Press [♦] and [F3 Graph] (right picture).

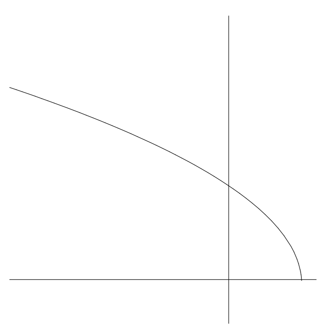




Now a question arises. Does the graph touch the $x-$axis? We have to determine the answer ourself, since the calculator graph cannot make it clear.

Find the $x-$intercept: $y = 0$ when $\sqrt{5 - 3x} = 0$ which happens when $5 - 3x = 0$, or $x = \dfrac{5}{3}$.

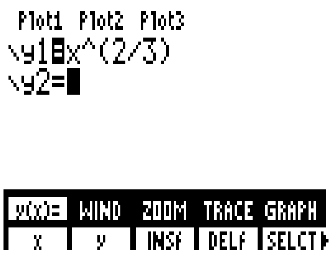
Thus, the point $\left(\dfrac{5}{3}, 0\right)$ *is* part of our graph. When we draw the graph on paper, we must connect the curve to the $x-$axis appropriately.

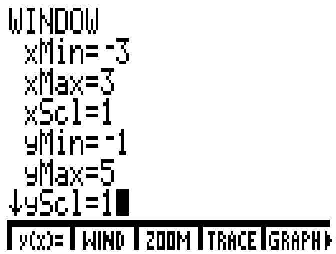


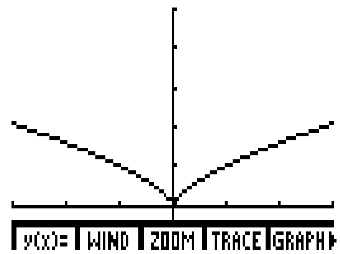
TI-86: Radical Functions

Graph $y = x^{2/3}$ with a domain of $(-\infty, \infty)$, using a window of $[-3, 3]$ by $[-1, 5]$.

Press Graph and F1 y(x)= . Clear any functions left over from a previous graph. Enter the function formula, using parentheses around the exponent (left picture). Press F2 Wind and enter values for $[-3, 3]$ by $[-1, 5]$ (middle picture). Press F5 Graph (right picture).



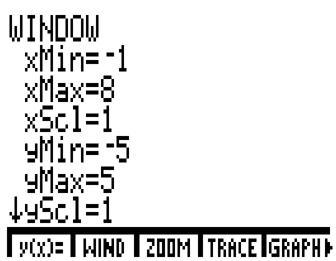


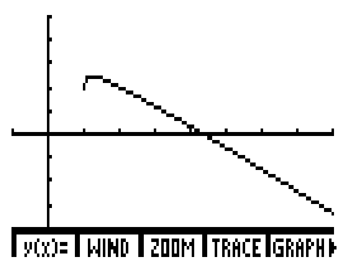


Notice, the point $(0, 0)$ is part of the graph, since it is the $x-$intercept.

Next, graph $y = \sqrt[4]{x-1} - x + 3$ with a domain of $[1, \infty)$, and a window of $[-1, 8]$ by $[-5, 5]$. Press F1 y(x)= . Press Clear to erase the previous function.

Enter the function formula, using the exponent of $\frac{1}{4}$ and parentheses appropriately (left picture). Press 2nd and F2 Wind . Set the window for $[-1, 8]$ by $[-5, 5]$ (middle picture). Press F5 Graph (right picture).



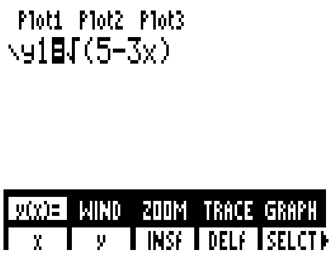


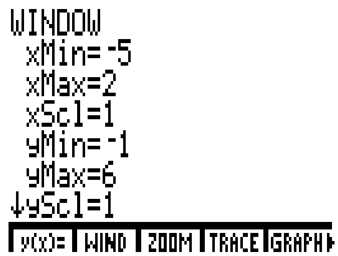
The domain is $[1, \infty)$. We also know when $x = 1$, $y = 2$, so the graph begins at the point $(1, 2)$.

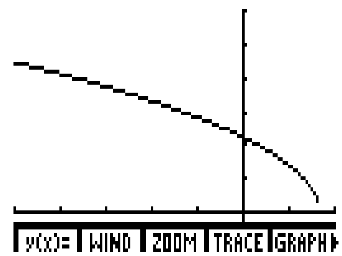
Finally, we graph $y = \sqrt{5-3x}$ with a domain of $\left(-\infty, \frac{5}{3}\right]$, and a window of $[-5, 2]$ by $[-1, 6]$.

Press F1 y(x)= . Press Clear to erase the previous function.

Enter the function formula and appropriate parentheses (left picture). Press 2nd and F2 Wind . Set the window for $[-5, 2]$ by $[-1, 6]$ (middle picture). Press F5 Graph (right picture).







Now a question arises. Does the graph touch the $x-$axis? We have to determine the answer ourself, since the calculator graph cannot make it clear.

Find the $x-$intercept: $y = 0$ when $\sqrt{5-3x} = 0$ which happens when $5 - 3x = 0$, or $x = \frac{5}{3}$. Thus, the point $\left(\frac{5}{3}, 0\right)$ *is* part of our graph. When we draw the graph on paper, we must connect the curve to the $x-$axis appropriately.