Chapter 9

Systems of Equations and Inequalities

Review sections as needed from Chapter 0, Basic Techniques, page 8.

9.1 Systems of Equations

Read/review section 0.3.3 (Circles, page 33).

Recall that to graph any equation on the calculator, we must first solve the equation for y.

For example, suppose we wish to solve graphically $\begin{cases} 2x^2 - y^2 &= -3 \\ x^2 + x + y^2 &= 6 \end{cases}$

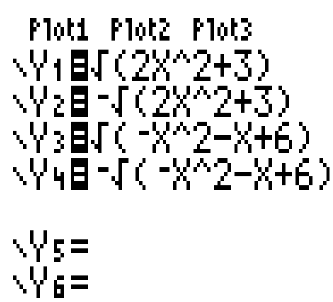
We must first solve each equation for y.

$2x^2 - y^2 = -3$ becomes $y^2 = 2x^2 + 3$ and $y = \pm\sqrt{2x^2+3}$

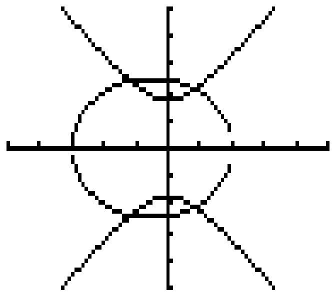
$x^2 + x + y^2 = 6$ becomes $y^2 = -x^2 - x + 6$ and $y = \pm\sqrt{-x^2-x+6}$

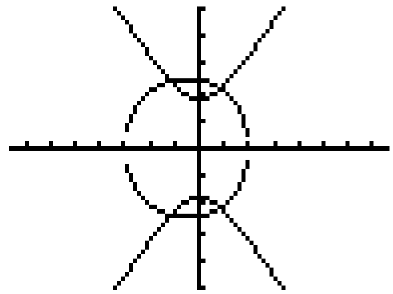
We should recognize the second equation $x^2 + x + y^2 = 6$ as a familiar shape. What is it?

So, we need to graph these functions.



Graphing all four functions with a window of $[-5, 5]$ by $[-5, 5]$, we have the left graph, or using zoom square, we get the right graph.





The curves $y_1 = \sqrt{2x^2 + 3}$ and $y_2 = \sqrt{2x^2 + 3}$, for the equation $2x^2 - y^2 = -3$, form a hyperbola (the parabola-looking curves turning up and down), while $y_3 = \sqrt{-x^2 - x + 6}$ and $y_4 = -\sqrt{-x^2 - x + 6}$, for the equation $x^2 + x + y^2 = 6$, form a circle.

We need to determine the four points of intersection in order to get the values of x and y that solve the system of equations.

The top two points of intersection are on the curves for y_1 and y_3. So, after we pick the intersection function, we have to indicate the first curve. y_1 is correct. To indicate the second curve, hit the down arrow key to get to curve y_3. (On the TI-83, you see the function definition in the upper left corner. On the TI-86 and TI-89, you see a number in the upper right corner indicating which definition is used, so we want definitions 1 and 3.) Continuing, give a guess if asked by moving the cursor closer to the actual point you are looking for, and press Enter.

We find the points of intersection, and thus two of the solutions, to be $(-1.18046, 2.4056129)$, $(.84712709, 2.106003)$.

Now find the other two solutions by finding the points of intersection for curves y_2 and y_4. Be careful to select the appropriate definitions for the function when using the intersection function. You select the definition needed by using the down arrow key.

We find the points of intersection, and thus the other two solutions, to be $(-1.18046, -2.405613)$, $(.84712709, -2.106003)$.

Therefore, our solutions to the system of equations, to two decimal places, are $(-1.18, 2.41)$, $(.85, 2.11)$, $(-1.18, -2.41)$, and $(.85, -2.11)$

Let's try one more of these systems of nonlinear equations. This time, it involves both y and y^2.

Solve graphically: $\begin{cases} x^2 - 2x &= 1 \\ x^2y^2 + xy &= 1 \end{cases}$

We can set the first equation equal to 0 and graph $y = x^2 - 2x - 1$.

To graph the second equation $x^2y^2 + xy = 1$, we need to solve for y.

So, get all terms to the left side $\quad x^2y^2 + xy - 1 = 0$.

This looks like a quadratic for the variable y. Since it is not factorable, we can use the quadratic formula to solve for y.
We have $\quad a = x^2$ (coefficient of y^2), $\quad b = x$ (coefficient of y), and $c = -1$.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-x \pm \sqrt{x^2 - 4(x^2)(-1)}}{2(x^2)} = \frac{-x \pm \sqrt{x^2 + 4x^2}}{2x^2}$$

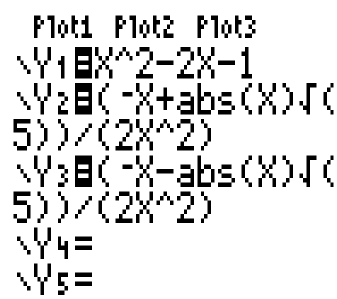
$$y = \frac{-x \pm \sqrt{5x^2}}{2x^2} = \frac{-x \pm |x|\sqrt{5}}{2x^2}$$

We may use either of these last two versions of the formula to get our graph.
Note the use of absolute value!

So, we may graph $\quad y = \dfrac{-x + \sqrt{5x^2}}{2x^2}$ and $y = \dfrac{-x - \sqrt{5x^2}}{2x^2}$, or

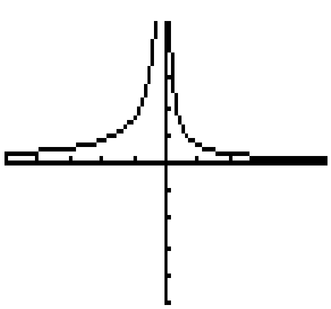
we may graph $\quad y = \dfrac{-x + |x|\sqrt{5}}{2x^2}$ and $y = \dfrac{-x - |x|\sqrt{5}}{2x^2}$.

Enter the three functions,
$y_1 = x^2 - 2x - 1, \;\; y_2 = \dfrac{-x + |x|\sqrt{5}}{2x^2}, \;\; y_3 = \dfrac{-x - |x|\sqrt{5}}{2x^2}$

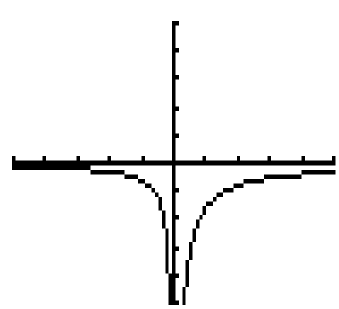


We know y_1 is a parabola, so let's see what y_2 and y_3 look like using a window of $[-5, 5]$ by $[-5, 5]$,

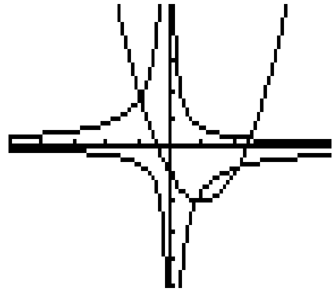
$y_2 = \dfrac{-x + |x|\sqrt{5}}{2x^2}$



$y_3 = \dfrac{-x - |x|\sqrt{5}}{2x^2}$



Graphing all three functions on the same graph, we have



As we can see, there are four points of intersection; thus, four solutions to our system of equations.

Graph y_1 intersects twice with y_2 (in quadrants I and II), and y_1 intersects twice with the right side of y_3 in quadrant IV.

Finding these four points (and four solutions), we have

$(-.9326049, 1.7349618)$, $(2.4991006, 2.4730257)$, $(.8220346, -1.968328)$ and $(2.1105703, -.7666335)$.

Therefore, our four solutions, using two decimal places, are

$(-.93, 1.73)$, $(2.50, 2.47)$, $(.82, -1.97)$ and $(2.11, -.767)$.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

By graphing, solve each system of equations, correct to two decimal places.

1. $$\begin{cases} x^2y = 2 \\ \ln x^2 = y \end{cases}$$

2. $$\begin{cases} xy^2 = 2 \\ y^4 = 2x+3 \end{cases}$$

3. $$\begin{cases} y^2 - 3xy = -2x^2 \\ y = |x^2-1| \end{cases}$$

4. $$\begin{cases} y^2 - 5xy = -2x^2 \\ y = \ln(-x+3) \end{cases}$$

5. $$\begin{cases} 2y^2 + xy - x^2 = 0 \\ xy + x + 6 = 0 \end{cases}$$

6. $\begin{cases} x^2 - 6x + y^2 + 2y = 1 \\ y = \dfrac{1}{x-3} \end{cases}$

7. $\begin{cases} (x+2)^2 + (y-1)^2 = 4 \\ y^2 - 2y - x - 5 = 0 \end{cases}$

8. $\begin{cases} (x-0.5)^2 + (y+2.1)^2 = 4 \\ y^2 + 4y - x + 1 = 0 \end{cases}$

9.2 Systems of Linear Equations in Two Variables

Caution: In some of the problems below, you will be graphing two lines on the same coordinate system and then finding the $x-$intercept for line y_1 and line y_2. When you use the *zero*, or *root*, function, be sure the correct line, y_1 or y_2, is selected for the *zero* you need.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

1. By graphing on your calculator, find the triangular region formed by $y = 4x - 0.8,\ 2x + y - 2.3 = 0$, and the $x-$axis.

   Find the area of this region, correct to one decimal place.

2. By graphing on your calculator, find the triangular region formed by $x - y = -1.3,\ x + y = 2.7$, and the $x-$axis.

   Find the area of this region, correct to one decimal place.

3. By graphing on your calculator, find the triangular region formed by $x - y = -1.57,\ x + y = 3.48$, and $y = -3$.

   Find the area of this region, correct to one decimal place.

4. By graphing on your calculator, find the triangular region formed by $2x - y = 1.4,\ 2x + y = -3.2$, and $y = 3$.

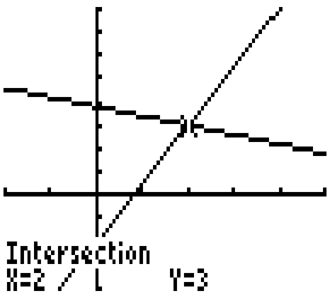
   Find the area of this region, correct to one decimal place.

5. Without graphing or solving the system to find the final answer, determine which graph is the most likely solution for this system of equations. Explain the reasoning for your choice. Hint: Consider the slope and $y-$intercept.

$$\begin{cases} 2x + 5y &= 19 \\ -5x + 2y &= -4 \end{cases}$$

   window $[-2, 5]$ by $[-4, 8]$

(A)



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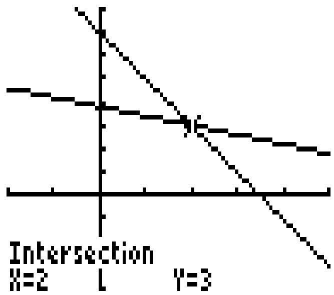

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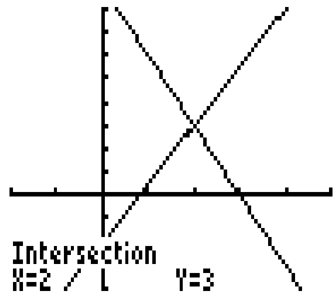
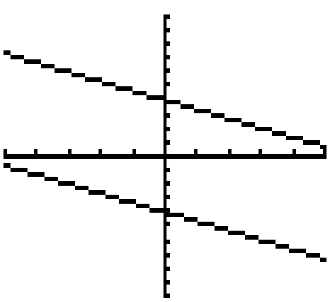


6. Without graphing or solving the system to find the final answer, determine which graph is the most likely solution for this system of equations. Explain the reasoning for your choice. Hint: Consider the slope and $y-$intercept.

$$\begin{cases} 2x - 3y &= 12 \\ -x + \dfrac{3}{2}y &= 6 \end{cases}$$

   window $[-5, 5]$ by $[-10, 10]$

(A)



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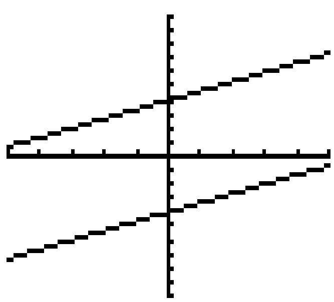

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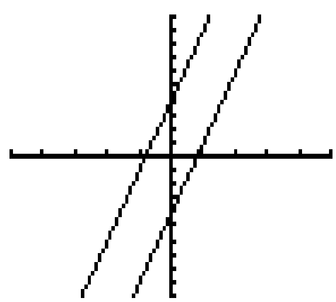
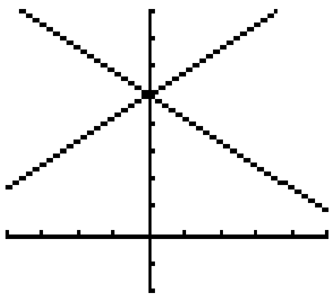


7. Without graphing or solving the system to find the final answer, determine which graph is the most likely solution for this system of equations. Explain the reasoning for your choice. Hint: Consider the slope and $y-$intercept.

$$\begin{cases} \dfrac{1}{2}x + \dfrac{3}{5}y & = & 3 \\ \dfrac{5}{3}x + 2y & = & 10 \end{cases}$$

window $[-4, 5]$ by $[-2, 8]$

(A)



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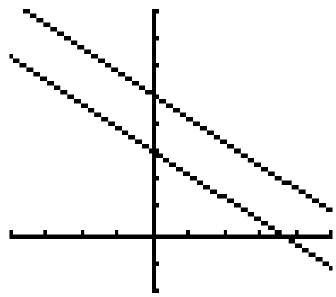

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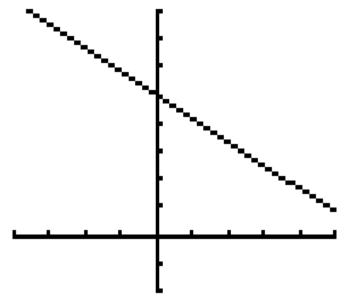

9.3 Systems of Linear Equations in Several Variables

See textbook for exercises.

9.4 Systems of Linear Equations: Matrices

Read section 0.12 (Solving Systems of Equations with Matrices), page 163.

See textbook for exercises.

9.5 The Algebra of Matrices

See textbook for exercises.

9.6 Inverses of Matrices and Matrix Equations

Read section 0.13 (Inverse Matrices), page 186.

See textbook for exercises.