

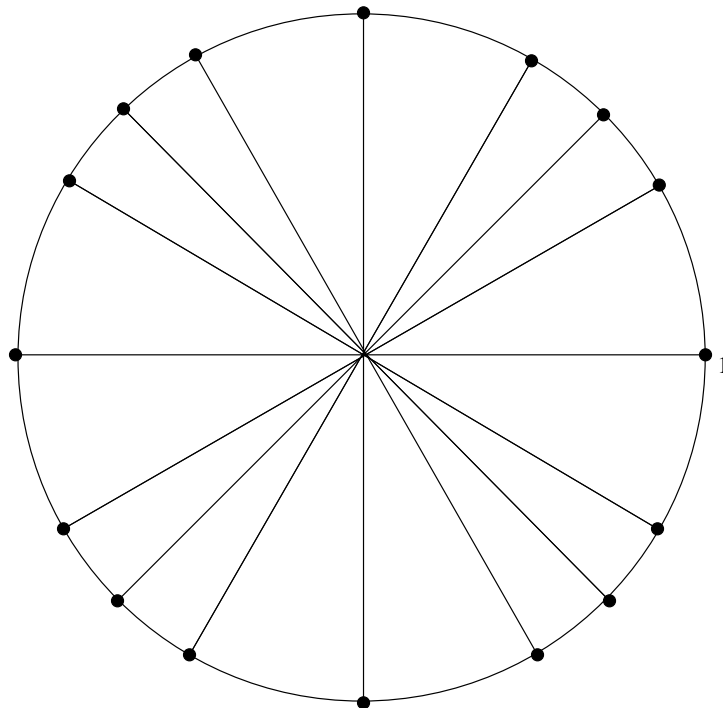
Chapter 5

Trigonometric Functions of Real Numbers

5.1 The Unit Circle

Exercises

1. The point where the line intersects the unit circle represents the terminal point for an arclength t drawn on the circle starting at the point $(1, 0)$. Label each terminal point shown on the circle between 0 and 2π , inclusive, with the appropriate real number, such as $\frac{\pi}{6}$, $\frac{2\pi}{3}$, $\frac{5\pi}{3}$, etc.

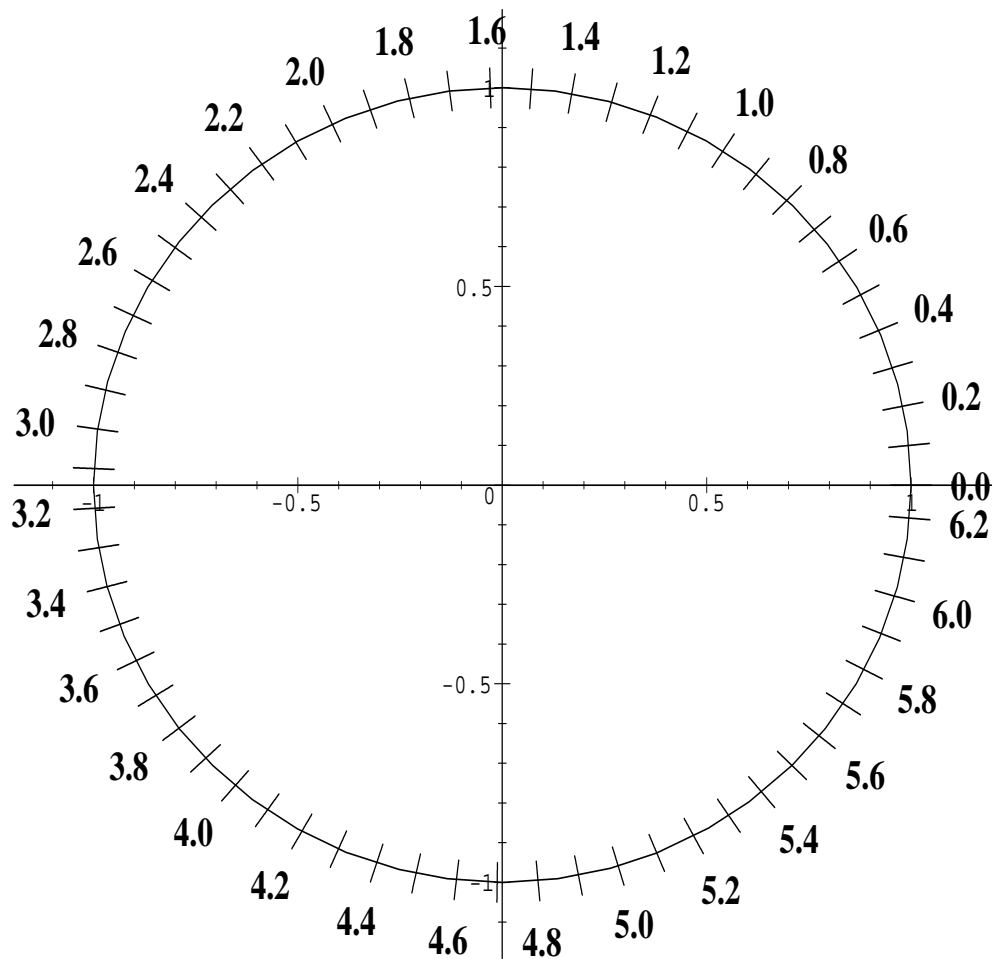


2. Find the terminal point determined by the real number t , with coordinates estimated to the closest one decimal place. Do not use a calculator for these.

(a) $t = 5.8$

(b) $t = -5.8$

(c) $t = -1.9$

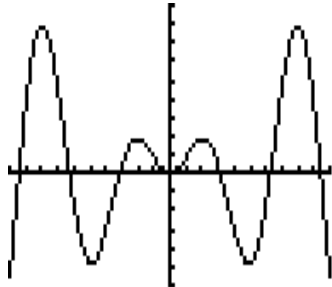


5.2 Trigonometric Functions of Real Numbers

Exercises

Determine whether the function represented by the graph is even, odd, or neither.

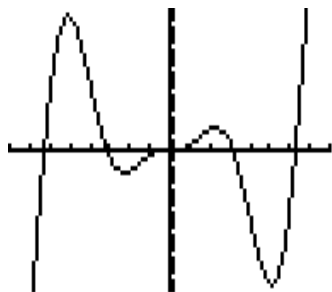
1.



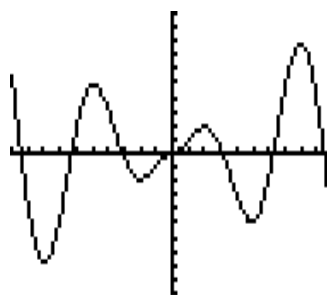
2.



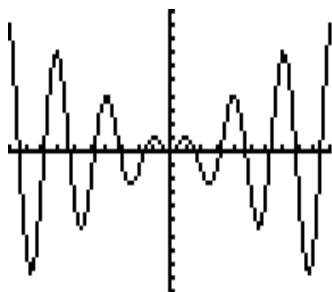
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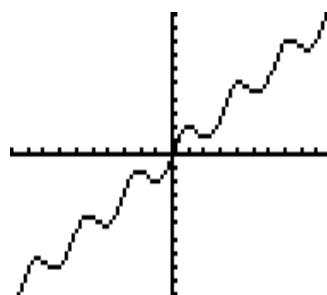
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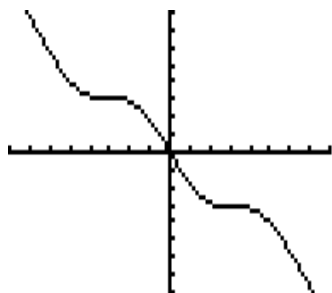
5.



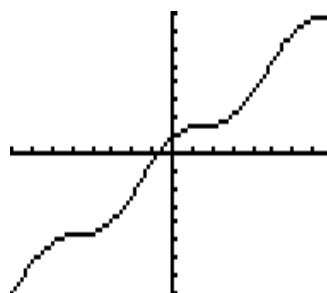
6.



7.



8.



9. Rewrite each of the following so $-x$ is no longer used, and simplify the resulting functions if possible.

(a) $\sin(-x) \cos(-x) \tan(-x)$

(b) $\sin^3(-x) \cos^3(-x) \sec^2(-x)$

10. Rewrite each of the following so $-x$ is no longer used, and simplify the resulting functions if possible.

(a) $\cos(-x) \tan(-x) \csc(-x)$

(b) $\sec(-x) \cot(-x) \tan(-x)$

11. Find the function requested, using exact values, no calculators. Show work.

Example: Find $\sec(-x)$ if $\sec(x) = -3$.

We know $\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos(x)} = \sec(x)$. So, we have $\sec(-x) = \sec(x) = -3$.

(a) Find $\sin(-x)$ if $\sin(x) = -\frac{1}{4}$.

(b) Find $\tan(-x)$ if $\tan(x) = -\sqrt{3}$.

(c) Find $\csc(-x)$ if $\csc(x) = 2$.

12. Find the function requested, using exact values, no calculators. Show work.

(a) Find $\cos(-x)$ if $\cos(x) = -\frac{1}{4}$.

(b) Find $\sec(-x)$ if $\sec(x) = 2$.

(c) Find $\cot(-x)$ if $\cot(x) = 1$.

5.3 Trigonometric Graphs

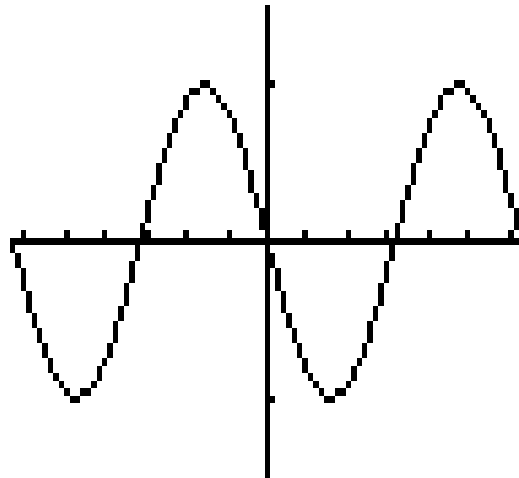
Read section 0.6.3 (Trigonometric Functions, page 75). When graphing trigonometric functions, be sure to set the calculator for radian mode. See section 0.6.3 (Trigonometric Functions page 75), or section 0.8.3 (Mode Settings, page 118) for placing the calculator into radian mode.

Exercises

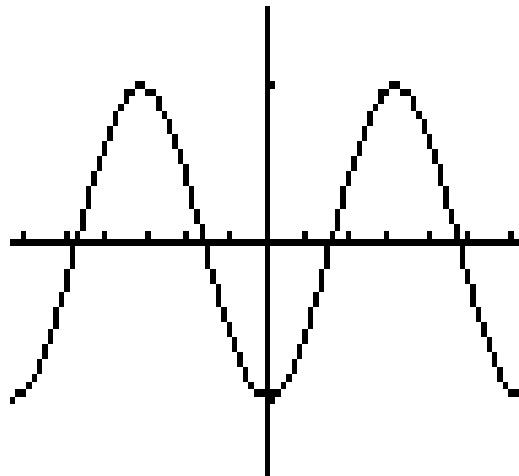
Label the x -intercept(s) and y -axis so the graph shown matches the graph of the function given. Clearly label the y -intercept when it is not zero.

In each case, state the amplitude, period, reflections if any, phase shift if any, to explain your choices for the x -intercepts and y -intercept.

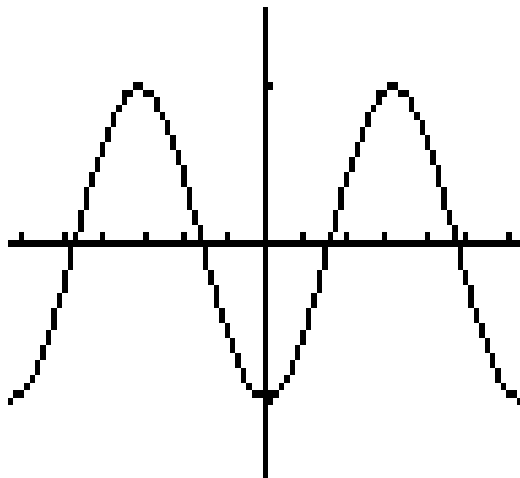
1. $y = -\sin(x)$



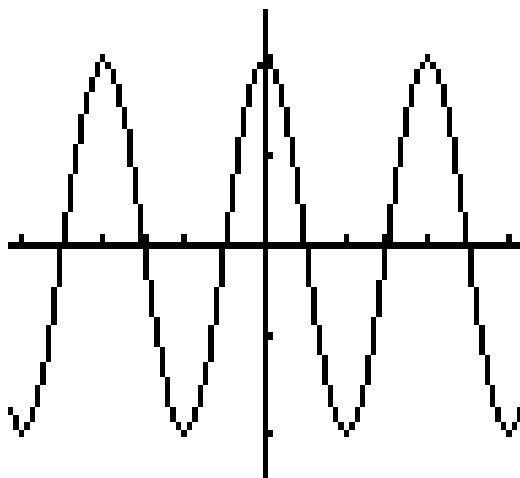
2. $y = \sin\left(x - \frac{\pi}{2}\right)$



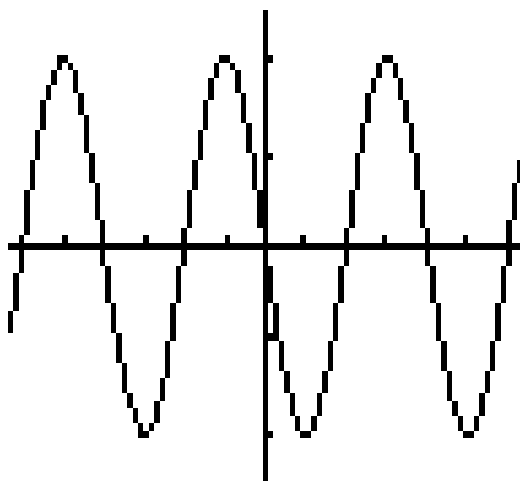
3. $y = \cos(x + \pi)$



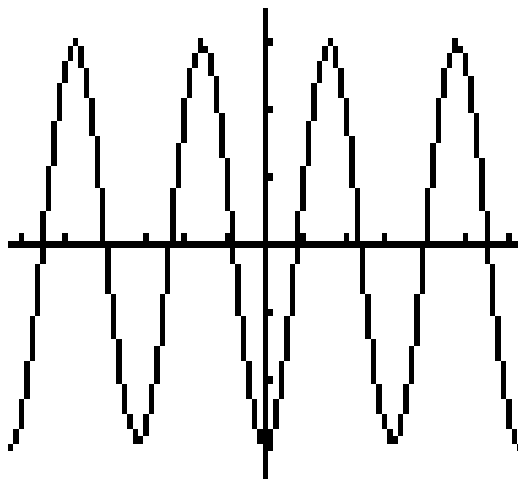
4. $y = 2 \cos\left(\frac{\pi}{2}x\right)$



5. $y = -2 \sin\left(\frac{\pi}{2}x\right)$

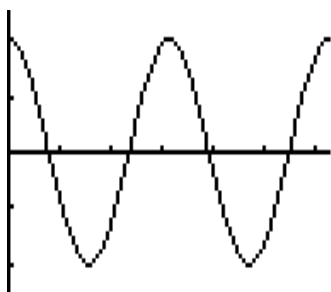


6. $y = -3 \cos(2x)$

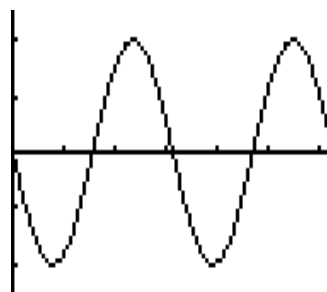


Match the given graph (A, B, C, D) to one of the functions shown in Exercises 7 - 12. If there is no graph for the function, state “no graph given”. In each case, explain your reasoning by indicating the amplitude, period, and any reflections. Be sure to take the window size into account when thinking about the period and the graph.

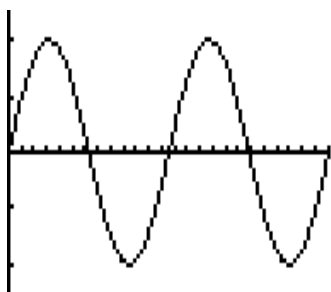
(A) window $[0, 2\pi]$ by $[-2.5, 2.5]$



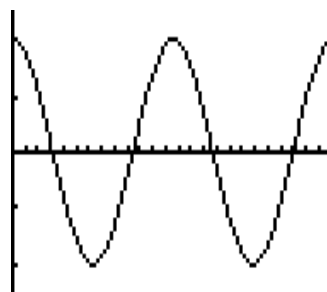
(B) window $[0, 2\pi]$ by $[-2.5, 2.5]$



(C) window $[0, 8\pi]$ by $[-2.5, 2.5]$



(D) window $[0, 8\pi]$ by $[-2.5, 2.5]$



7. $y = 2 \cos\left(\frac{1}{2}x\right)$

8. $y = -2 \sin(2x)$

9. $y = -2 \cos(2x)$

10. $y = 2 \cos(2x)$

11. $y = 2 \sin\left(\frac{1}{2}x\right)$

12. $y = -2 \sin\left(\frac{1}{2}x\right)$

See 0.4.1 (Graphing a Family of Functions, page 46), to review family of functions notation,

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

When graphing trigonometric functions whose period involves a multiple of π , use a window which uses an appropriate multiple of π for the x -values. For example, if the period is 2π , using x -values of $[-2\pi, 2\pi]$ will graph two complete periods of the function. Indicate the window x -values as $[-2\pi, 2\pi]$ instead of $[-6, 6]$. Remember to use radian mode.

13. Given $f(x) = A \sin(x)$

- (a) Using a window of $[-2\pi, 2\pi]$ by $[-3, 3]$, graph $f(x)$ for $A = -3, -1$, all on the same graph. Sketch the graphs for reference.
- (b) Using a window of $[-2\pi, 2\pi]$ by $[-3, 3]$, graph $f(x)$ for $A = 1, 2, 3$, all on the same graph. Sketch the graphs for reference.
- (c) How do the x -intercepts change, if any, as A changes?
- (d) How does the amplitude change as A changes?
- (e) How does the period change as A changes?

14. Given $f(x) = \cos(Bx)$

- (a) Using a window of $[-\pi, \pi]$ by $[-2, 2]$, graph $f(x)$ for $B = 1, 2, 3$, on three different graphs, sketching each one for reference.
- (b) How does the amplitude change as B changes?
- (c) How many periods appear in each graph in the viewing window?
- (d) How does the period change as B changes?
- (e) Based on your observations, how many periods will appear in the given viewing window $[-\pi, \pi]$ for the graph $y = \cos(nx)$, for a positive integer n ?

15. $f(x) = 3 \sin(120x)$

(a) Determine an appropriate window for graphing $f(x)$ for exactly two periods. Show some algebraic work to indicate how you found the period. Indicate the window using exact values, using some multiple of π for the x -values when appropriate.

(b) Graph $f(x)$ on the calculator for two periods.

16. $f(x) = \cos(30x)$

(a) Determine an appropriate window for graphing $f(x)$ for exactly two periods. Show some algebraic work to indicate how you found the period. Indicate the window using exact values, using some multiple of π for the x -values when appropriate.

(b) Graph $f(x)$ on the calculator for two periods.

Graph each of the following functions. From the graph, determine if the function is periodic. If so, find the period by using the graph. Express the period in terms of π when appropriate. (Remember, $\pi \approx 3.14159$, $2\pi \approx 6.28318$)

In some cases, the graph on the calculator may not appear to have x -intercepts when, with some thinking and checking, we know it does touch the x -axis. So, do not take the graph totally at face-value without some additional thinking. Show in your answer what you think the correct version of the graph would be, including the appropriate x -intercepts.

17. $y = |\sin(x)|$

18. $y = 2^{\cos(x)}$

19. $y = \cos|x|$

20. $y = \sin|x|$

5.4 More Trigonometric Graphs

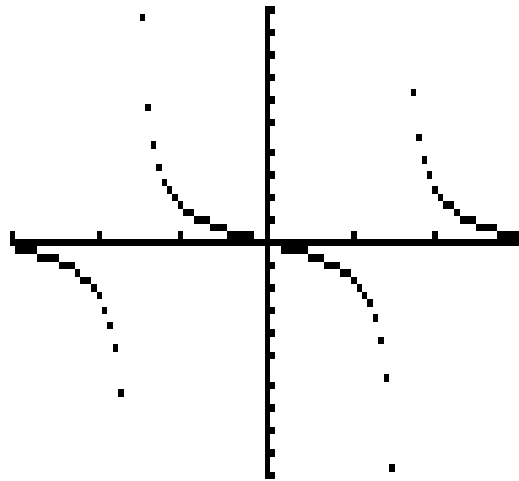
Exercises

Label each graph with the following so the graph shown matches the graph of the given function.

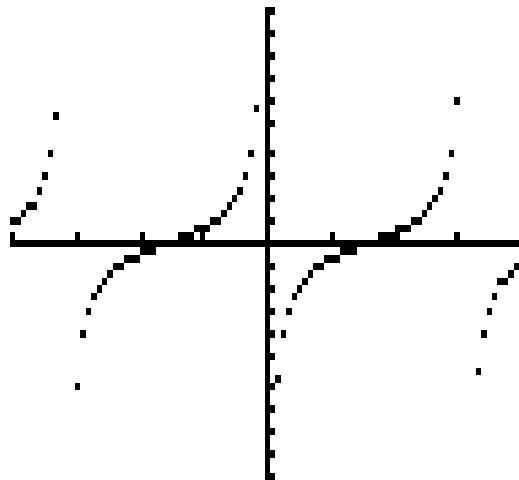
- the x -intercept(s) for the tangent and cotangent graphs,
- the y -axis for the secant and cosecant graphs,
- the y -intercept, if it is not zero,
- the vertical asymptotes (and draw the asymptotes as dashed lines).

In each case, state the period, phase shift if any, reflections if any, to explain your choices.

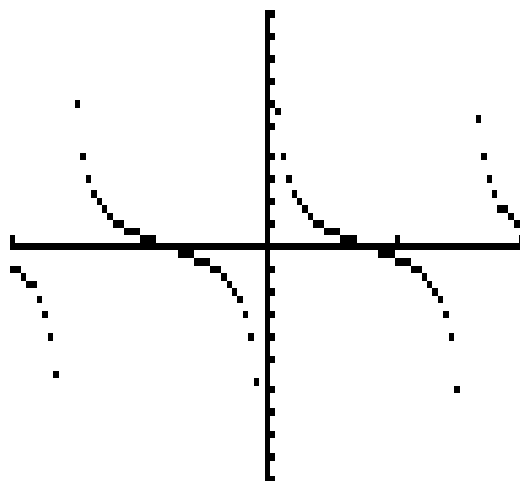
1. $y = -\tan(x)$



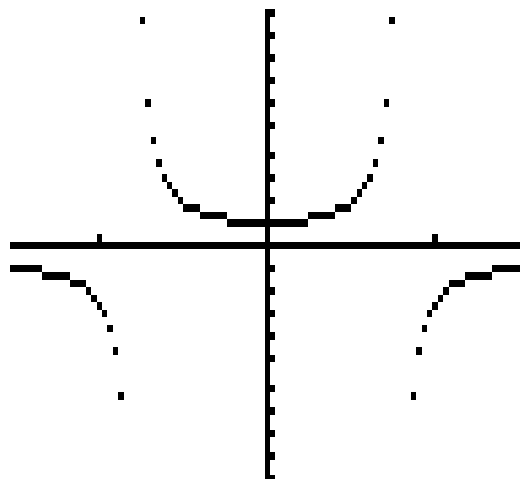
2. $y = \tan\left(x + \frac{\pi}{2}\right)$



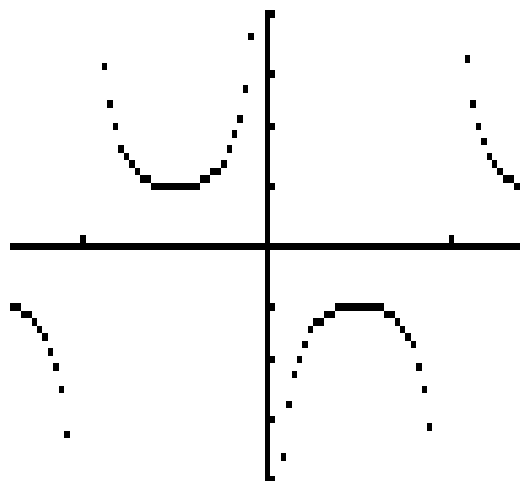
3. $y = \cot(2x)$



4. $y = \sec(2x)$

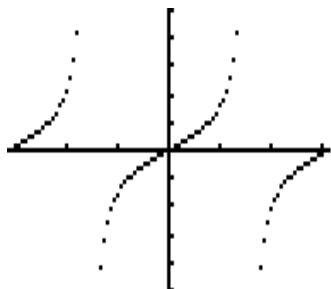


5. $y = -\csc(\pi x)$

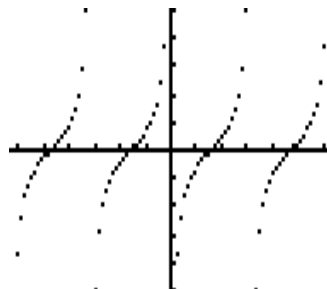


Match the given graph (A, B, C, D, E, F) to one of the functions shown in Exercises 6 - 14. If there is no graph for the function, state "no graph given". In each case, explain your reasoning by indicating the period, any phase shift, any reflection, and asymptotes. Be sure to take the window size into account when thinking about the period and the graph.

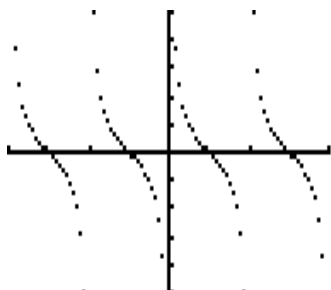
(A) window $[-\pi, \pi]$ by $[-5, 5]$



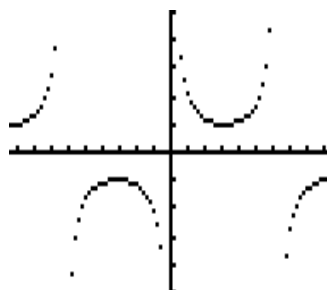
(B) window $[-2\pi, 2\pi]$ by $[-5, 5]$



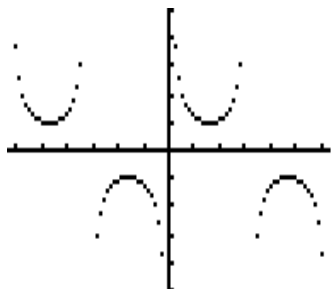
(C) window $[-2, 2]$ by $[-5, 5]$



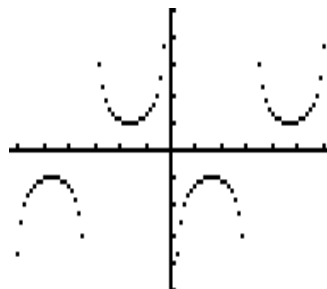
(D) window $[-3\pi, 3\pi]$ by $[-5, 5]$



(E) window $[-2\pi, 2\pi]$ by $[-5, 5]$



(F) window $[-2\pi, 2\pi]$ by $[-5, 5]$



6. $y = -\cot(x)$

7. $y = \sec\left(x - \frac{\pi}{2}\right)$

8. $y = \tan(x + \pi)$

9. $y = \csc(2x)$

10. $y = \csc\left(\frac{1}{2}x\right)$

11. $y = -\csc(x)$

12. $y = \cot(\pi x)$

13. $y = \tan(\pi x)$

14. $y = \tan(x - \pi)$

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor. Remember to use radian mode.

15. $f(x) = \tan(25x)$

(a) Determine an appropriate window for graphing $f(x)$ for exactly two periods. Show some algebraic work to indicate how you found the period. Indicate the window using exact values, using some multiple of π for the x -values when appropriate.

(b) Graph $f(x)$ on the calculator for two periods.

16. $f(x) = \tan(10\pi x)$

(a) Determine an appropriate window for graphing $f(x)$ for exactly two periods. Show some algebraic work to indicate how you found the period. Indicate the window using exact values, using some multiple of π for the x -values when appropriate.

(b) Graph $f(x)$ on the calculator for two periods.

Graph each of the following functions. From the graph, determine if the function is periodic. If so, find the period by using the graph. Express the period in terms of π when appropriate.

17. $y = |\tan(x)|$

18. $y = \cot|x|$

19. $y = \sec|x|$

20. $y = |\csc(x)|$

5.5 Modeling Harmonic Motion

See textbook for exercises.