Chapter 4

Exponential and Logarithmic Functions

Review sections as needed from Chapter 0, Basic Techniques, page 8.

4.1 Exponential Functions

Read section 0.6.1 (Exponential Functions, page 68).

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

<u>*Exercises*</u>

In Exercises 1 and 2,

(a) State the transformations needed to create $g(x), h(x)$ and $k(x)$ from $f(x)$.

(b) Write the formula for each function using the basic function given.
For example, if $f(x) = 2^x$, and we have $g(x) = f(x+1)$, then we have $g(x) = 2^{(x+1)}$.

(c) Graph each function $g(x), h(x)$ and $k(x)$ by hand using transformations. No calculators.

(d) Graph on the calculator your formula for $g(x), h(x)$ and $k(x)$ from part (b).
If the calculator graph does not match your hand graph, determine where the mistake is and correct it.

1. $f(x) = 3^x$

   $g(x) = f(x-1), \quad h(x) = f(-x) + 2, \quad k(x) = -f(x) - 1$

2. $f(x) = e^x$

   $g(x) = -f(x+1), \quad h(x) = -f(-x), \quad k(x) = f(x-1) + 2$

3. Which function in each of the following grows faster as $x \to \infty$? Graph these on your calculator to help you decide.

   (a) $f(x) = 2^x, \quad g(x) = x^{2.5}$

   (b) $f(x) = 3^x, \quad g(x) = x^{3.7}$

4. For each function shown, before graphing, determine the domain and range (write answer in interval notation). Think about these and see what you can figure out about the exponential functions. Then graph the function on your calculator and compare the graph with your domain and range.

   (a) $f(x) = 3^{\frac{1}{x}}$ What is the behavior of y as $x \to 0^-$?

   (b) $f(x) = 3^{\frac{1}{x^2}}$ What is the behavior of y as $x \to \infty$?

   (c) $f(x) = 3^{x^2}$

5. Graph $f(x) = 2^{x-x^2}$. Round answers to two decimal places.

   On what intervals is $f(x)$ increasing ?

   On what intervals is $f(x)$ decreasing?

   What is the maximum value?

6. Graph $f(x) = 2^{1-x-x^2}$. Round answers to two decimal places.

   On what intervals is $f(x)$ increasing?

   On what intervals is $f(x)$ decreasing?

   What is the maximum value?

7. Graph $f(x) = x^2\left(2^{-x}\right)$. Round answers to two decimal places.

   On what intervals is $f(x)$ increasing?

   On what intervls is $f(x)$ decreasing?

   What are the local maximum and minimum values?

8. Graph the function requested and answer these questions.

   On what intervals is the function increasing?

   On what intervals is the function decreasing?

   What are the local maximum and minimum values? Specify whether each value is a maximum or minimum.

   Explain why you think the graph looks as it does considering what you know about the graph of $y = e^x$.

   (a) $f(x) = e^{|x|}$ (b) $g(x) = e^{-|x|}$

9. Approximate the real zeros, to two decimal places, for $f(x) = 1 - x^2 + 2^{-x}$.

10. Simplify algebraically, then graph the original expression and the simplified expression on your calculator to verify they are the same.

    $$e^{-x}(e^x + 1) - (e^{-x} + 1)(e^x - 1)$$

11. Simplify algebraically, then graph the original expression and the simplified expression on your calculator to verify they are the same.

    $$e^{-x}\left(e^x + 2\right) - \left(2e^x + 1\right)\left(e^{-x} - 2\right)$$

4.2 Logarithmic Functions

Read section 0.6.2 (Logarithmic Functions, page 71).

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

1. Decide on the domain, then graph each function on your calculator.

   (a) $y = e^{\ln x}$

   (b) $y = \ln(e^x)$

   (c) Simplify the functions in parts (a) and (b). Explain why (a) and (b) do not have the same domain and graph.

2. Simplify $f(x) = e^{\ln \pi}$.

   Graph $f(x)$ without a calculator, then do it with the calculator.

3. Find the domain algebraically, expressing answer in interval notation.

   Graph the function on the calculator. Compare the domain from the graph with your algebraic answer. If they do not agree, figure out why.

   When drawing the graph from the calculator, be sure to include the appropriate asymptote as a dashed line. (Does the graph actually stop where the calculator indicates?)

   (a) $f(x) = \log(5 - 2x)$

   (b) $f(x) = \ln |x|$

4. Find the domain algebraically, expressing answer in interval notation.

   Graph the function on the calculator. Compare the domain from the graph with your algebraic answer. If they do not agree, figure out why.

   When drawing the graph from the calculator, be sure to include the appropriate asymptote as a dashed line. (Does the graph actually stop where the calculator indicates?)

   (a) $f(x) = \log(x^2 - 2x)$

   (b) $f(x) = \ln(x) + \ln(3 - x)$

   (c) $f(x) = \dfrac{1}{\ln(x)}$

5. Given $f(x) = e^x - 2$ and $g(x) = \ln(x + 1)$, by graphing, determine the following, to two decimal places, and use interval notation appropriately.

   (a) For what values of x is $f(x) = g(x)$?

   (b) For what values of x is $f(x) < g(x)$?

6. Given $f(x) = e^x$ and $g(x) = x^4$, by graphing, determine the following, to two decimal places, and use interval notation appropriately.

   (a) For what values of x is $f(x) = g(x)$

   (b) For what values of x is $f(x) < g(x)$

7. Find the maximum/minimum values for each of the following.
   Specify whether it is a maximum or minimum value.

   Find intervals where the function is increasing or decreasing, correct to two decimal places.

   (a) $y = 6x - 2x\ln(2x)$

   (b) $y = x^2(\ln(x) - 2)$

8. Find the maximum/minimum values for each of the following.
   Specify whether it is a maximum or minimum value.

   Find intervals where the function is increasing or decreasing, correct to two decimal places.

   (a) $y = x^2(2 - \ln(x))$

   (b) $y = \ln(2x) - x^2 + 3$

9. $f(x) = e^x$ and $g(x) = \ln(x)$ are inverses of each other. Graph $f(x)$ and $g(x)$ on the same set of axes along with the dashed line $y = x$. Explain how the graphs of f(x) and g(x) support the fact that f(x) and g(x) are inverse functions,

10. We usually see logarithmic graphs using a base of $a > 1$. What happens if $0 < a < 1$? Do the following.

    (a) Graph $f(x) = (\frac{1}{2})^x$ by hand.

    (b) Graph $f^{-1}(x) = \log_{\frac{1}{2}}(x)$ by reflecting $f(x)$ over the line $y = x$.

11. Graphically, compare $f(x) = \log(x)$, $g(x) = \sqrt{x}$, and $h(x) = x$, where $1 < x \leq 16$. Put the functions in order from largest to smallest as $x \to \infty$.

4.3 Laws of Logarithms

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

1. Graph $y = \log\left(\frac{x}{2}\right)$, $y = \frac{\log(x)}{\log(2)}$ and $y = \log(x) - \log(2)$.
   Which two are the same?

2. Graph $y = \log(x - 2)$, and $y = \log(x) - \log(2)$.
   Are they the same? Should they be?

3. Graph $y = \Big(\log(3)\Big)^x$ and $y = x\log(3)$.
   Are they the same? Should they be?

4. Graph $y = \ln(2^x)$ and $y = x\ln(2)$.
   Are they the same? Should they be?

4.4 Exponential and Logarithmic Equations

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

Solve graphically, correct to four decimal places, using interval notation where appropriate.

1. $\ln(x-2)+\ln(x^2) > 1.5$
2. $\ln(x^2-3) > \sqrt{5}$
3. $\ln(x)+\log(x) = 2.5$
4. $\ln(x+2) = \log(1-x)$
5. $e^{2x} = x+3$
6. $e^{2x} - 3e^{x} = 3$
7. $\ln(1.4-x) = (x+2.23)^2 - 1$
8. $\ln(x^2-x+1) = e^{x^2-3}$

When given $a < b < c$, we can graph $y_1 = a, y_2 = b, y_3 = c$ and read our answer from the graph to determine when $a < b < c$, or $y_1 < y_2 < y_3$. Use this idea to solve for x in exercises 9 and 10.

9. $2 \leq 2.16\ln(5-x) \leq 4$
10. $-1 \leq 5.23\ln(x+3.2) \leq 8$

4.5 Modeling with Exponential and Logarithmic Functions

See the textbook for exercises.