Chapter 2

Functions

2.1 What is a Function?

Review section 0.3.4 (Radicals, page 38), section 0.3.1 (Rational Functions, page 21).

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

Find the domain of each function, using interval notation for the answer
a) algebraically (no calculator),
b) graphing the function (with calculator) and reading the information from the graph to check your answer. If your answers do not match, try to find your error and fix it.

Remember, on some of the graphs the curves may not appear to touch the $x-$axis when we "know" the curves actually do have $x-$intercepts. Find the $x-$intercepts if needed to verify the curve touches the $x-$axis.

1. $f(x) = \sqrt{1-x^2}$

2. $f(x) = \sqrt{x^2-1}$

3. $f(x) = \sqrt[3]{7-3x}$

4. $f(x) = \sqrt{5-3x}$

5. $f(x) = \dfrac{2x+1}{x-2}$

6. $f(x) = \dfrac{5}{5-2x}$

7. $f(x) = -2$

8. $f(x) = 3$

9. $f(x) = \sqrt{\dfrac{x-2}{1-x}}$

10. $f(x) = \sqrt{\dfrac{1-x}{x^2+1}}$

11. $f(x) = \sqrt{2x^2+5x-3}$

12. $f(x) = \sqrt{2x^2+x-3}$

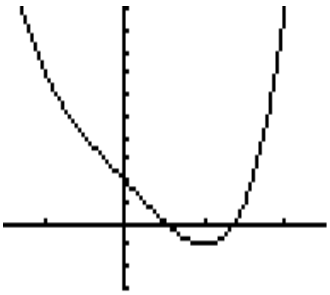
2.2 Graphs of Functions

Note: Number 77 on page 169 of the textbook does not work on the TI-89 if we use the "family of functions" notation. $f(x)$ must be graphed individually for each value of c given.

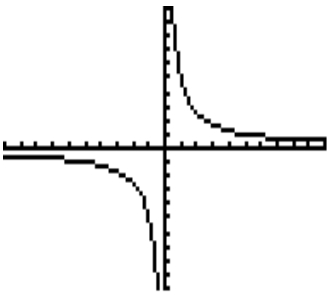
Exercises

Find the domain and range of each function by reading the needed information from the graph, using interval notation for the answers.

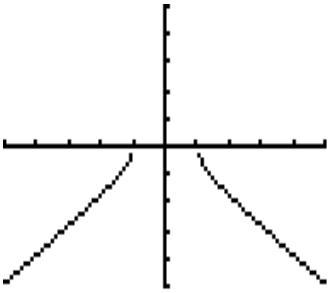
1) window $[-1.5, 2.5]$ by $[-3, 10]$



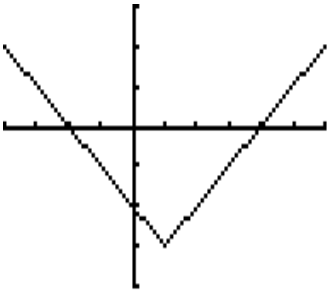
2) window $[-10, 10]$ by $[-10, 10]$



3) window $[-5, 5]$ by $[-5, 5]$
Assume curves touch the $x-$axis.



4) window $[-4, 6]$ by $[-4, 3]$



Exercises continued on the next page.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

For exercises 5 and 6, review section 0.4.1 (Graphing a Family of Functions, page 46).

5) On your calculator, graph $f(x) = x^2$ using $[-4, 4]$ by $[-1, 5]$, and draw it on paper. Now start over and graph (on the same coordinate system) the family of functions $f(x) = (x - c)^2$ for $c = -2, -.5, 1, 2$ using the same window as above. How does the value of c affect the graph?

6) On your calculator, graph $f(x) = \sqrt{x}$ using $[-1, 5]$ by $[-3, 5]$, and draw it on paper. Now start over and graph (on the same coordinate system) the family of functions $f(x) = \sqrt{x} + c$ for $c = -2, -1, 1, 2$ using the same window as above. How does the value of c affect the graph?

For exercises 7 - 10, read section 0.5 (Piecewise-defined Functions, page 56).

Graph each piecewise-defined function using your calculator. Remember to clearly indicate whether the endpoint is included, •, or not included, ∘, when you copy the graph to paper. (Find the coordinates as if you are graphing $f(x)$ by hand, and use • or ∘ appropriately. Do *not* use trace to find the points, it will not work correctly.)

7) $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

8) $f(x) = \begin{cases} 3 - x^2 & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$

9) $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \leq x \leq 2 \\ -x + 6 & \text{if } x > 2 \end{cases}$

10) $f(x) = \begin{cases} 3 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x < 2 \\ -2 & \text{if } x \geq 2 \end{cases}$

11) Given $|x^2 - 2| = k$, find appropriate values for k, if such exist, for which this equation has 0 solutions, 1 solution, 2 solutions, 3 solutions, and 4 solutions. (Try graphing $y = |x^2 - 2|$ and see how this may help.)

12) Given $|x^3 - 3x + 2| = k$, find appropriate values for k, if such exist, for which this equation has 0 solutions, 1 solution, 2 solutions, and 3 solutions. (Try graphing $y = |x^3 - 3x + 2|$ and see how this may help.)

2.3 Increasing and Decreasing Functions; Average Rate of Change

Read/review 0.7 (Graphing Features, page 83) to find the minimum and maximum values in order to more easily find the high and low points when needed on the graphs below.

Read/review section 0.5 (Piecewise-defined Functions, page 56).

Review section 0.3.2 (Absolute Functions, page 28) if needed.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

Graph each function using the calculator and determine the intervals where the function is increasing and decreasing, rounding to two decimal places. On the piecewise-defined functions, be sure to indicate on the graph if the endpoint is included or not, using • or ∘ appropriately, as well as include the endpoint or not appropriately for the increasing/decreasing intervals.

1. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$

2. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$

3. $f(x) = 0.4x^3 - 0.6x^2 - 3x - 2$

4. $f(x) = -0.4x^4 - 0.2x^3 + 0.8x^2 - 2$

5. $f(x) = \begin{cases} 3.2 - x & \text{if } x \le 1.2 \\ 2.1x - 1.5 & \text{if } x > 1.2 \end{cases}$

6. $f(x) = \begin{cases} 2.3x - 1.2 & \text{if } x < 1.4 \\ -0.4x - 0.8 & \text{if } x \ge 1.4 \end{cases}$

7. $f(x) = \begin{cases} 1.1x + 2 & \text{if } x < -1 \\ 1.1x & \text{if } -1 \le x \le 2 \\ -1.1x - 1 & \text{if } x > 2 \end{cases}$

8. $f(x) = \begin{cases} -1.6x + 1 & \text{if } x \le -1 \\ x - 1.2 & \text{if } -1 < x \le 3 \\ -x + 1 & \text{if } x > 3 \end{cases}$

9. $f(x) = \left|3x^2 - 2x + 1\right| - 5$

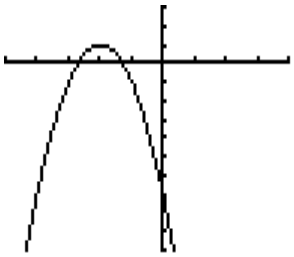
10. $f(x) = 5 - \left|3x^3 - 3.7x^2 + 5x - 4\right|$

2.4 Transformations of Functions

Exercises

1. Is each function even, odd, or neither?

a)



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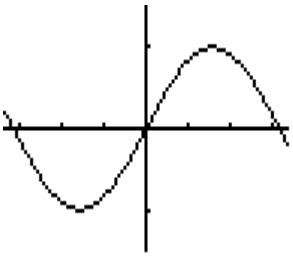

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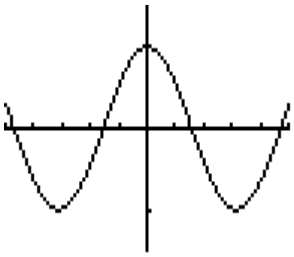
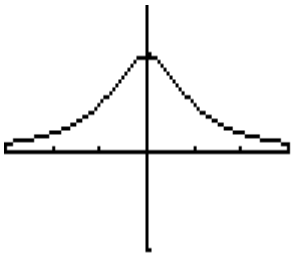


d)



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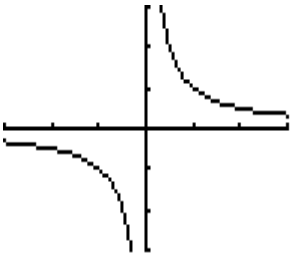
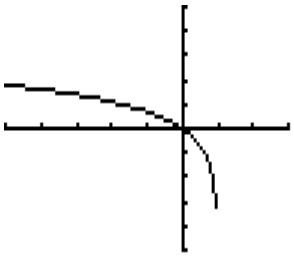
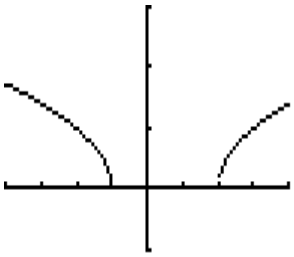


f)



2. Is each function even, odd, or neither?

a)



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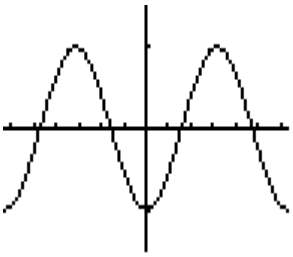

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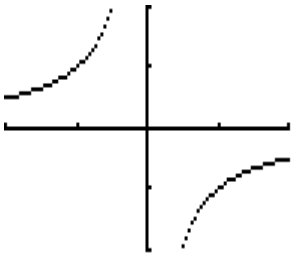

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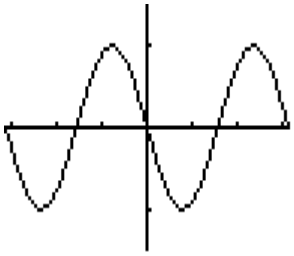

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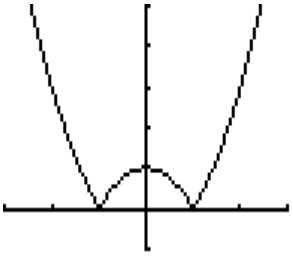

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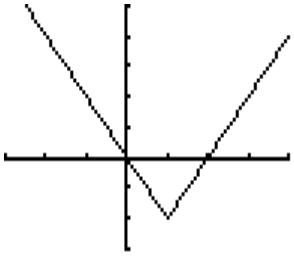


Graph each function with your calculator and determine if it is even, odd, or neither.

Review section 0.3.2 (Absolute Functions, page 28) if needed.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

3. $f(x) = -0.5x^4 + 2x^2 - 1$

4. $f(x) = x^3 + x^2 + 3$

5. $f(x) = 0.75x^2 + |x| + 1$

6. $f(x) = -x^5 + 2x^3$

7. Graph on your calculator $f(x) = x$ and $g(x) = |x|$ on two separate coordinate systems, copying the graphs to paper. In comparing the two graphs, what conclusion can you reach about how the graphs change in looking from $f(x) = x$ to $g(x) = |x|$? (So, how did the absolute value change $f(x)$?)

8. Graph on your calculator $f(x) = x^2 - 3$ and $g(x) = |x^2 - 3|$ on two separate coordinate systems, copying the graphs to paper. In comparing the two graphs, what conclusion can you reach about how the graphs change in looking from $f(x) = x^2 - 3$ to $g(x) = |x^2 - 3|$? (So, how did the absolute value change $f(x)$?)

9. Do the following:
   a) Graph on your calculator $f(x) = x^3 + x^2 - 6x$.
   b) Sketch the graph on paper.
   c) Using the graph of part (b), sketch on paper the graph of $|f(x)| = |x^3 + x^2 - 6x|$.
   d) Finally, graph $|f(x)| = |x^3 + x^2 - 6x|$ on your calculator to check your sketch of part (c).

10. Do the following:
   a) Graph on your calculator $f(x) = 0.5x^4 - 2x^2 + 1$.
   b) Sketch the graph on paper.
   c) Using the graph of part (b), sketch on paper the graph of $|f(x)| = |0.5x^4 - 2x^2 + 1|$.
   d) Finally, graph $|f(x)| = |0.5x^4 - 2x^2 + 1|$ on your calculator to check your sketch of part (c).

11. Given $f(x) = 1.2x^2 - 0.8x - 1.7$, state the transformations needed for each of the following. Graph $f(x)$ and each function given on your calculator to see if the graph changes as you expected. Use the same window for all graphs.

   a) $g(x) = f(x + 2) - 1$ b) $h(x) = -f(x - 1)$ c) $j(x) = f(-x)$

12. Given $f(x) = -x^2 - 3.1x + 1.8$, state the transformations needed for each of the following. Graph $f(x)$ and each function given on your calculator to see if the graph changes as you expected. Use the same window for all graphs.

a) $g(x) = -f(x-2)$ b) $h(x) = f(x+1) - 3$

2.5 Quadratic Functions: Maxima and Minima

Read/review section 0.7 (Graphing Features, page 83) for directions on finding the minimum and maximum of functions on your calculator.

See the textbook for exercises.

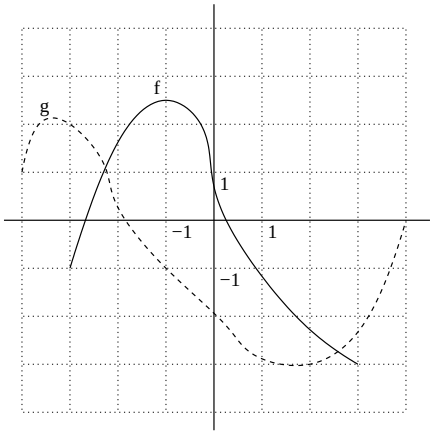
2.6 Modeling with Functions

See the textbook for exercises.

2.7 Combining Functions

Exercises

1. Use the graph given and find the domain of each function, in interval notation, estimating values as needed.

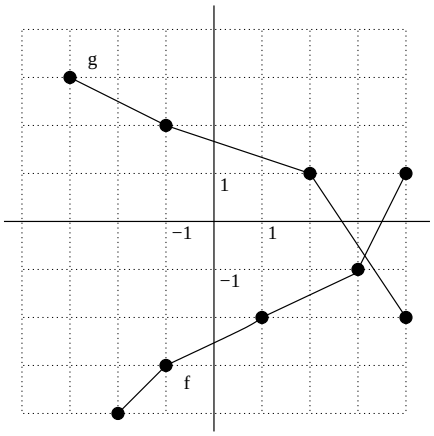


a) $(f+g)(x)$

b) $\left(\frac{f}{g}\right)(x)$

c) $\left(\frac{g}{f}\right)(x)$

2. Use the graph given and find the domain of each function, in interval notation, estimating values as needed.

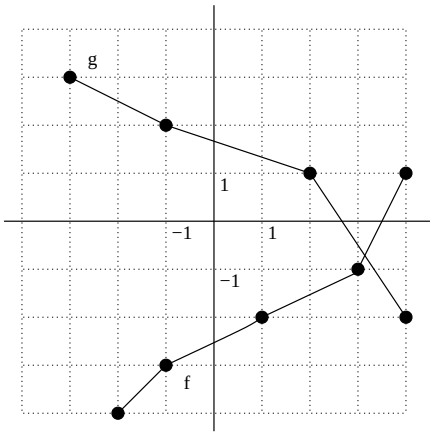


a) $(f+g)(x)$

b) $\left(\frac{f}{g}\right)(x)$

c) $\left(\frac{g}{f}\right)(x)$

3. Using this graph:

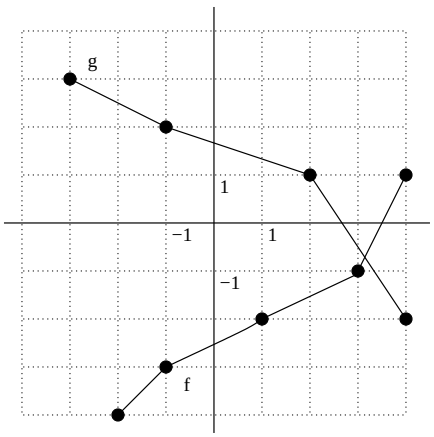


a) find $f(g(-3))$

b) find x such that $g(f(x)) = 2$

c) find x such that $f(f(x)) = -2$

4. Using this graph:



a) find $g(f(-1))$

b) find x such that $f(g(x)) = -4$

c) find x such that $g(g(x)) = 1$

5. Given $f(x) = \dfrac{2}{x+a}$ and $g(x) = \sqrt{x-5}$ for appropriate real numbers a,

algebraically find the domain of $(f \circ g)(x)$. Express answer in words (or using inequalities), not interval notation. Your answer will involve a in some form.

6. Given $f(x) = \sqrt{x+a}$ and $g(x) = 2x - 1$ for appropriate real numbers a,

algebraically find the domain of $(f \circ g)(x)$. Express answer in words (or using inequalities), not interval notation. Your answer will involve a in some form.

2.8 One-to-One Functions and Their Inverses

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

Exercises

1. Algebraically find the inverse, $f^{-1}(x)$, for $f(x) = 2x + 3$. Then graph $f(x), f^{-1}(x)$, and the line $y = x$ on the same coordinate system. Notice the symmetry around the line $y = x$.

2. Algebraically find the inverse, $f^{-1}(x)$, for $f(x) = x^3 - 2$. Then graph $f(x), f^{-1}(x)$, and the line $y = x$ on the same coordinate system. Notice the symmetry around the line $y = x$.

3. Graph $f(x) = \sqrt[3]{x^3+4} - 2, g(x) = \sqrt[3]{x^3+2} - 4, h(x) = \sqrt[3]{(x+2)^3 - 4}$ on the same coordinate system.
   (a) Which two functions appear to be symmetric about the line $y = x$?
   (b) Use the Inverse Function Property to verify these two functions are inverses of each other.

4. Graph $f(x) = \sqrt[3]{x^3+2} - 3, g(x) = \sqrt[3]{x^3+3} - 2, h(x) = \sqrt[3]{(x+2)^3 - 3}$ on the same coordinate system.
   (a) Which two functions appear to be symmetric about the line $y = x$?
   (b) Use the Inverse Function Property to verify these two functions are inverses of each other.